Math 4740

9/18/23

Test 1 moved to Monday 10/9

was scheduled for 10/2

CA Superlotto Plus

A ticket consists of:

- 5 "lucky" numbers chosen from 1-47
- 1 "mega" number chosen from 1-27

- No repeats in the lucky numbers. But the mega number can be the same as a lucky number.
- Order of lucky #s doesn't matter. It's always in numerical order on the ticket.

Example tickets:

$\underbrace{7 \quad 13 \quad 18 \quad 23 \quad 40}_{\text{lucky \#s}} \quad \underbrace{23}_{\text{mega\#}}$ ← ex ticket #1

$$\boxed{\underbrace{1 \quad 2 \quad 3 \quad 4 \quad 5}_{\text{lucky \#s}} \qquad \underbrace{6}_{\text{mega \#}}} \leftarrow \text{ex ticket \#2}$$

How many possible tickets are there?
If you want to think of a sample space of all possible tickets:

$$S = \{ \underbrace{(\{7, 13, 18, 23, 40\}, 23)}_{\text{ticket 1}}, \underbrace{(\{1, 2, 3, 4, 5\}, 6)}_{\text{ticket 2}}, \underbrace{\ldots}_{\text{lots more}} \}$$

How many possible tickets?

$$\underbrace{\binom{47}{5}}_{\text{\# of ways to pick 5 lucky \#s from 1-47}} \cdot \underbrace{\binom{27}{1}}_{\text{\# ways to pick 1 mega \# from 1-27}}$$

$$= \frac{47!}{5!(47-5)!} \cdot 27$$

$$= \frac{47!}{5!\,42!} \cdot 27$$

$=$

Fact:

$$\binom{n}{1} = \frac{n!}{1!(n-1)!} = \frac{n \cdot (n-1)!}{(n-1)!} = n$$

$8! = 8[7!]$

That is, $\binom{n}{1} = n$

$$= \frac{47\cdot \cancelto{23}{46}\cdot \cancelto{3}{\cancelto{9}{45}}\cdot \cancelto{11}{44}\cdot 43\cdot \cancel{(42!)}}{(\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1)\cancel{(42!)}}\cdot 27$$

$$= 47\cdot 23\cdot 3\cdot 11\cdot 43\cdot 27$$

$$= \boxed{41{,}416{,}353 \text{ possible tickets}}$$

<u>Q:</u> What is the probability that if you buy one ticket you will get the 5 lucky #s correct and the mega # correct?

<u>A:</u> $\dfrac{1}{41{,}416{,}353} \approx 0.00000002414\ldots$

$\approx 0.000002414\,\%$

Q: What $\overbrace{\text{are the odds}}^{\text{is the probability}}$ of getting exactly 3 of the 5 lucky #s and not the mega # ?

#'s drawn by the magical lottery machine

$$\underbrace{3 \quad 12 \quad 15 \quad 41 \quad 42}_{\text{lucky \#s}} \quad \underbrace{17}_{\text{mega}}$$

How many tickets will get exactly 3 of the 5 lucky #s and not the mega ?

you want your ticket in this group

$\boxed{47-5=42}$

$$\underbrace{\binom{5}{3}}_{\text{choose 3 of the 5 winning}} \cdot \underbrace{\binom{42}{2}}_{\text{choose 2 non-winning}} \cdot \underbrace{\binom{26}{1}}_{\text{not picking winning}} =$$

lucky #s	lucky #s	mega #
Ex: 3, 15, 42	1, 7	1
12, 41, 42	43, 45	12
⋮	⋮	⋮

$$\frac{5!}{3!(5-3)!} \cdot \frac{42!}{2!(42-2)!} \cdot 26$$

$$= \frac{5!}{3!\,2!} \cdot \frac{42!}{2!\,40!} \cdot 26$$

$$= \frac{120}{(6)(2)} \cdot \frac{42 \cdot 41 \cdot \cancel{(40!)}}{(2)\cancel{(40!)}} \cdot 26$$

$$= (10)(861)(26)$$

$$= \boxed{223{,}860 \text{ tickets}}$$

$$\text{Probability} = \frac{223{,}860}{41{,}416{,}353}$$

$$\approx 0.00540511\ldots$$

$$\approx \boxed{0.540511\ \%}$$

lottery website says the probability is

$$\frac{1}{185} \approx 0.00540541\ldots$$

Ex: Suppose five 6-sided dice are rolled. What is the probability that exactly two of the dice have 6's showing?

Ex:

6	1	2	6	4
die 1	die 2	die 3	die 4	die 5

Sample space size:

$$\frac{6 \text{ possibilities}}{\text{die 1}} \quad \frac{6 \text{ possibilities}}{\text{die 2}} \quad \frac{6 \text{ possibilities}}{\text{die 3}} \quad \frac{6 \text{ possibilities}}{\text{die 4}} \quad \frac{6 \text{ possibilities}}{\text{die 5}}$$

$$= 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$$

$$= \boxed{7,776}$$

How many rolls have exactly two 6's?

Step 1: Choose two of the dice to get the two 6's.

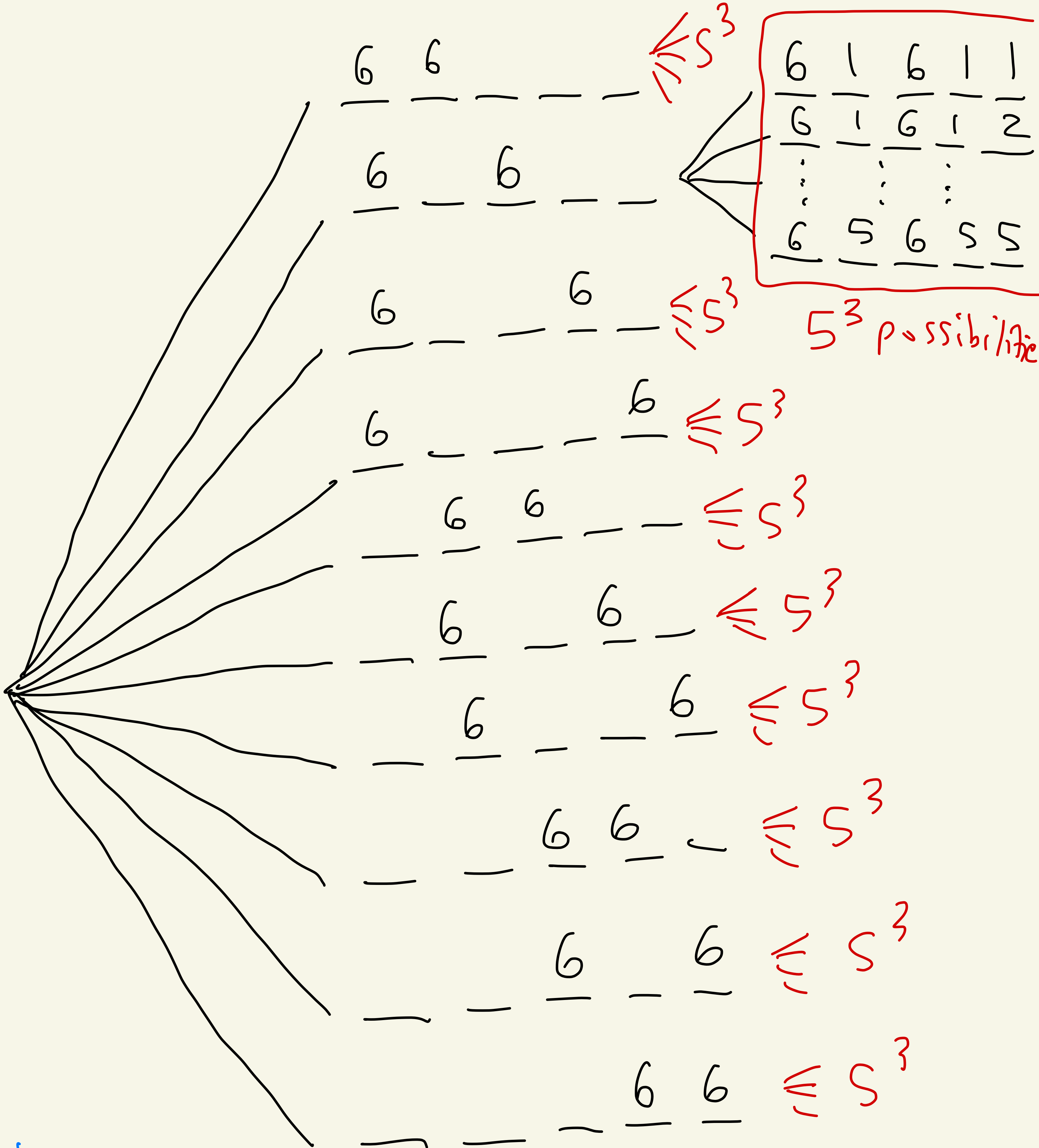
There are $\binom{5}{2} = 10$ ways to do this.

$$\underset{\text{die 1}}{\underline{\ 6\ }}\quad \underset{\text{die 2}}{\underline{\quad}}\quad \underset{\text{die 3}}{\underline{\ 6\ }}\quad \underset{\text{die 4}}{\underline{\quad}}\quad \underset{\text{die 5}}{\underline{\quad}}$$

Step 2: Fill in the non-6's.

$$\underset{\text{die 1}}{\underline{\ 6\ }}\quad \underset{\text{die 2}}{\underline{\begin{matrix}5\\ \text{choices}\end{matrix}}}\quad \underset{\text{die 3}}{\underline{\ 6\ }}\quad \underset{\text{die 4}}{\underline{\begin{matrix}5\\ \text{choices}\end{matrix}}}\quad \underset{\text{die 5}}{\underline{\begin{matrix}5\\ \text{choices}\end{matrix}}}$$

There are 5^3 ways to do this.



Step 1: $\binom{5}{2} = 10$ possibilities

Step 2: 5^3

Answer:

$$\frac{10 \cdot 5^3}{7{,}776}$$

$$\approx 0.16075...$$

$$\approx 16\%$$

chance you get exactly two 6's.