Math 4740
$9 / 13 / 23$

Ex: In how many ways can 5 people be seated in a row of 5 seats?

Example seating

$$
\frac{M}{\text { seat }} \frac{B}{\substack{\text { Seat } \\ 2}} \frac{C}{\substack{\text { sent } \\ 3}} \frac{S}{\substack{\text { Seat } \\ 4}} \frac{D}{\frac{5}{\text { seat }}}
$$

Ben

$$
\begin{aligned}
& \text { Answer: }
\end{aligned}
$$

$$
\begin{aligned}
& =5!=120
\end{aligned}
$$

Ex: Suppose we have 3 math books and 2 biology books. How many ways can we put the books on a shelf so that the math books are next to each other?

Ex:


| Math | Bio |
| :--- | :--- |
| Calculus <br> Probability <br> Algebra | Evolution |



Another way to count:
Step 1
Pick one of these $\underset{B 1}{\rightarrow} \rightarrow \sum_{\text {math }}^{B 2}$

3 possibilities in step 1
step 2 Fill in the books

$$
\begin{aligned}
& \underbrace{3 \cdot 2 \cdot 1}_{\substack{\text { Fill in } \\
\text { math }}} \cdot \underbrace{2 \cdot 1}_{\text {Fill in }_{\text {bio }}^{2}}=\sqrt{12 \text { possiniliks }} \begin{array}{l}
\text { in step 2 }
\end{array}] \\
& \text { Total }=3 \cdot 12=36
\end{aligned}
$$

(step) $\frac{1}{\operatorname{sitep} 2}$

Another way:
Think of math as a unit and two bio as seperate. So, 3 objects.
Step 1: Order the 3 objects


Step 2: Fill in math chunk math

32
3! ways to do this $3!=6$

$$
\text { Answer }=\frac{6 \cdot 6=36}{\substack{\uparrow \\ \frac{1}{s t e p 1} \\ s+e 0^{2}}}
$$

Suppose we have $n$ objects where $n$, of them are alike (ie the same or indistinguishable), $n_{2}$ of them are alike, $\ldots, n_{r}$ are alike where $n=n_{1}+n_{2}+\cdots+n_{r}$ Then there are

$$
\frac{n!}{n_{1}!n_{2}!\cdots n_{r}!}
$$

permutations of these objects

Ex: How many permutations are there of the letters


Combinations
Consider a set of size $n$. The number of subsets of size $r$ where $0 \leq r \leq n$ is

read:
"n chooser"

(This is the same as the \# of ways that r objects can be selected/chosen from $n$ objects where order doesn't matter

Ex:
Suppose a dealer has the following cards:

$$
A^{D} A^{P} Q^{\overrightarrow{2}} B^{\infty}
$$

How many ways can the dealer deal you two lands from these four?

$$
\text { Ex: } A^{8} A^{Y}<\begin{gathered}
\text { same }_{\text {as }}^{A^{Q} / A^{8}} \\
\text { order doesnt } \\
\text { matter }
\end{gathered}
$$



Ex: A dealer has a standard 52-cand deck.
They deal you 5 curds.
How many possible hands can you yet?
Ex hand:


Royal Flush!

Answer:

$$
\begin{aligned}
& \begin{aligned}
\binom{52}{5}
\end{aligned} \\
& =\frac{52!}{5!(52-5)!} \\
& \\
& =\frac{52!}{5!47!}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{13 \cdot 17}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!} \\
& (5 \cdot 96 \cdot 3 \cdot 2 \cdot 1)(47!
\end{aligned}
$$

