Math 4740

9/11/23

Topic 2 - Counting and probability

Basic counting principle

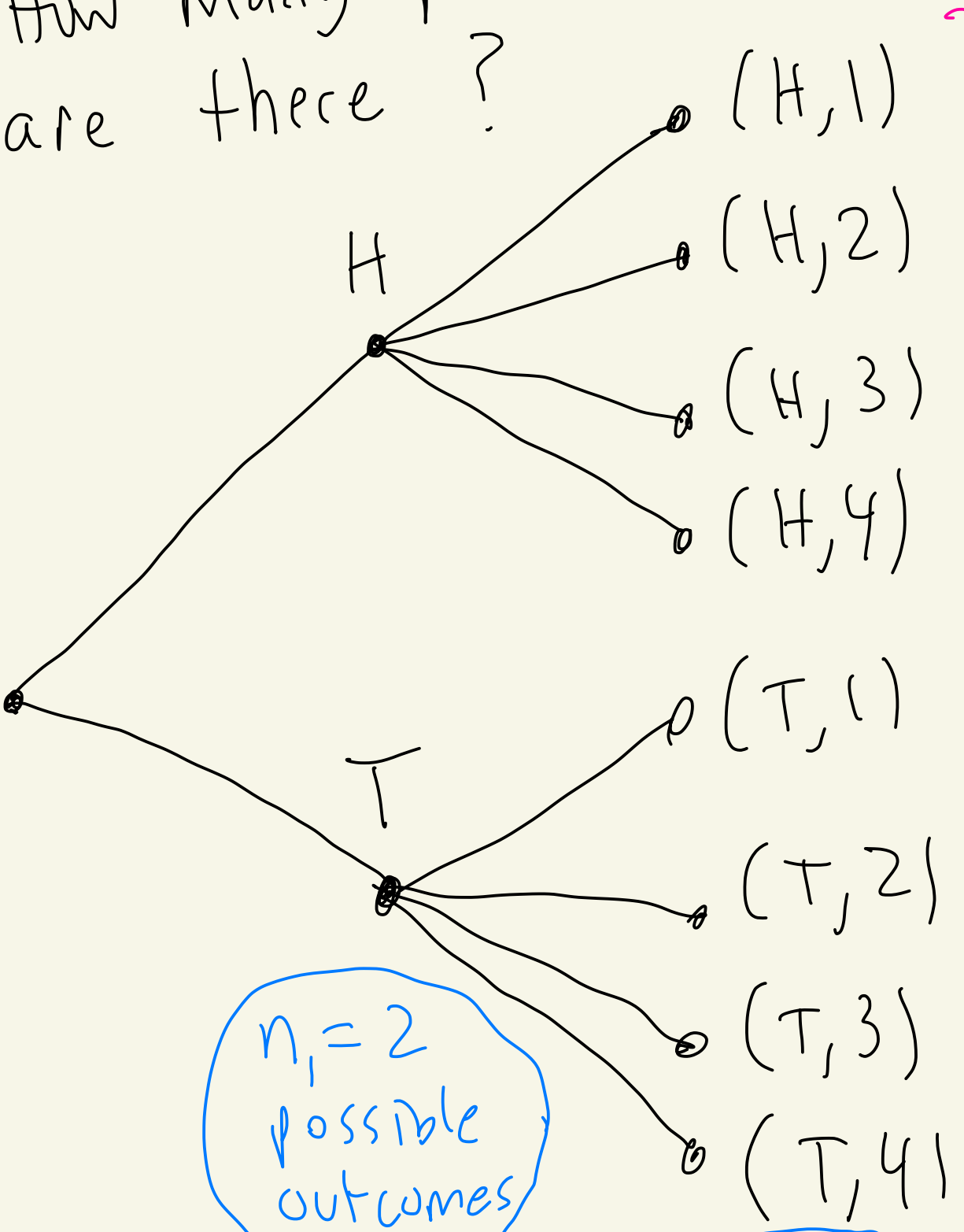
If r experiments are performed in a row such that the first experiment may result in n_1 possible outcomes; and if for each of these n_1 possible outcomes there are n_2 possible outcomes for the second experiment; and if for each of the possible outcomes of the first two experiments there are n_3 possible outcomes of the

third experiment, and if, ...,
then there are

$$n_1 \cdot n_2 \cdot n_3 \cdots n_r$$

possible outcomes for the
r experiments.

Ex: Suppose we toss a coin and then roll a 4-sided die how many possible outcomes are there?



Another way to write:

means H or T

means 1, 2, 3, or 4

$$\underline{H/T} \quad \underline{1,2,3,4}$$

$$2 \cdot 4 = 8$$

possibilities · possibilities

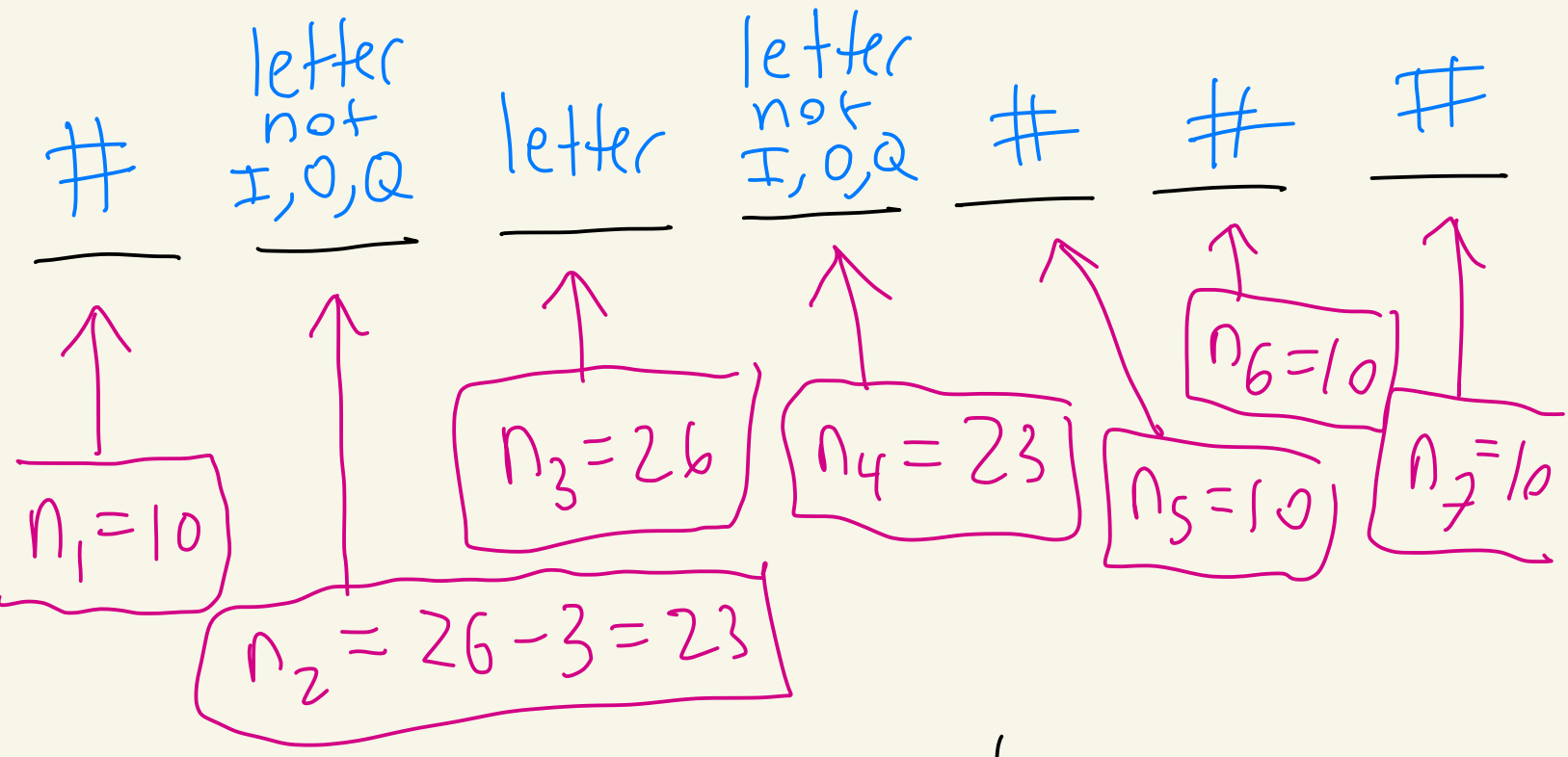
Ex: In California, a license plate consists of one number (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) followed by three upper-case letters, followed by three numbers. The only exclusion is that the letters I, O, and Q are not used in spot 2 and spot 4.

Examples are:

5 K A T 9 9 2

3 A Q A 1 2 3

How many possible license plates are there?



total # of possible license plates is

$$10 \cdot 23 \cdot 26 \cdot 23 \cdot 10 \cdot 10 \cdot 10$$

$$= 137,540,000$$

Birthday Paradox

Suppose there are N people in a classroom. What are the odds (probability) that there are at least two people with the same birthday? (This means month & day, not necessarily year. Such as at least two people born on 9/4)

Assumptions:

(1) We will assume that no one has a Feb 29

leap year birthday.

(2) We will assume that each day is equally likely

(3) Assume $N \leq 365$ because if $N > 365$ then the probability is 100%

Let's figure out the sample space.

What if $N = 3$?

$$S = \{(\text{date } 1, \text{date } 2, \text{date } 3) \mid \text{date } i \text{ is a calendar day}\}$$

$= \{$ (April 1, May 10, Feb 3),
student 1 student 2 student 3

(Jan 17, Oct 5, July 4),
student 1 student 2 student 3

ex of two with same bday → (Jan 15, Oct 3, Jan 15),
student 1 student 2 student 3

$\ldots \}$

Then, $|S| = 365 \cdot 365 \cdot 365$

$= (365)^3$

For general N, the size of the sample space is $(365)^N$

365 possibilities · 365 possibilities · · · · 365 possibilities

student 1 student 2 student N

Let E be the event that there are at least two people with the same birthday.

This is too hard to count.

So instead we count $\overline{E}$ which is the event that no one has the same birthday

Let's count the size of $\overline{E}$

365 possibilities · 364 possibilities · 363 possibilities · · · 365-(N-1)

student 1 student 2 student 3 student N

Cant have same bday as student 1

Cant have same bday as student 1 or student 2

cant have same bday as previous N-1 students

So,

$$|\bar{E}| = \underbrace{365 \cdot 364 \cdot 363 \cdots (365-(N-1))}_{\frac{365!}{(365-N)!}}$$

← will get to this later

Thus,

$$P(E) = 1 - P(\bar{E})$$

thm last week

$P(E)$: our goal

$$= 1 - \frac{|\bar{E}|}{|S|}$$

assumed every day equally likely

$$= 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365 - N + 1)}{(365)^N}$$

When $N = 3$ you get

$$P(E) = 1 - \frac{365 \cdot 364 \cdot 363}{(365)^3}$$

$$\approx 0.0082 \approx 0.82\%$$

N	P(E)
1	0%
2	0.274%
3	0.82%
4	1.64%

5	2,71 %
⋮	⋮
10	11.7 %
⋮	
18	34,7 %
⋮	
24	53,83 %
⋮	
40	89,12 %
⋮	
50	97.04 %

Permutations

Suppose you have n objects.
A permutation of those n objects is an ordered list of the n objects.

Ex: What are all the permutations of a, b, c?

permutations:

another way:

a b c ← (a, b, c)

a c b ← (a, c, b)

b a c ← (b, a, c)

b c a ← (b, c, a)

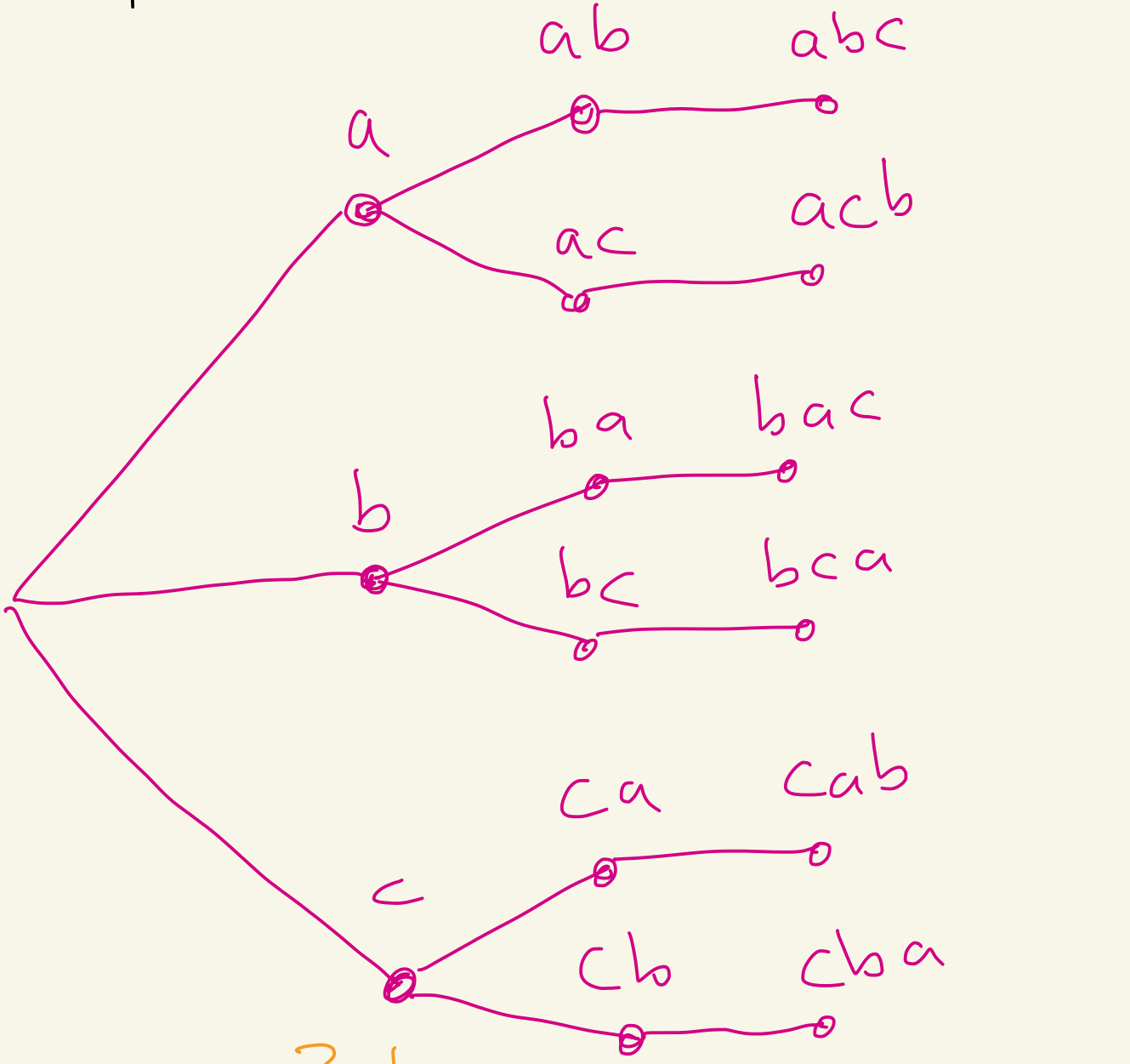
c a b ← (c, a, b)

c b a ← (c, b, a)

simpler way

math way to make order matter

6 possible permutations-



3 choices · 2 choices · 1 choice

3 choices · 2 choices · 1 choice

$3 \cdot 2 \cdot 1 = 3!$ possibilities

In general, there are $n!$ permutations of n objects

$n \quad n-1 \quad n-2 \cdots 1$
