Math 4740

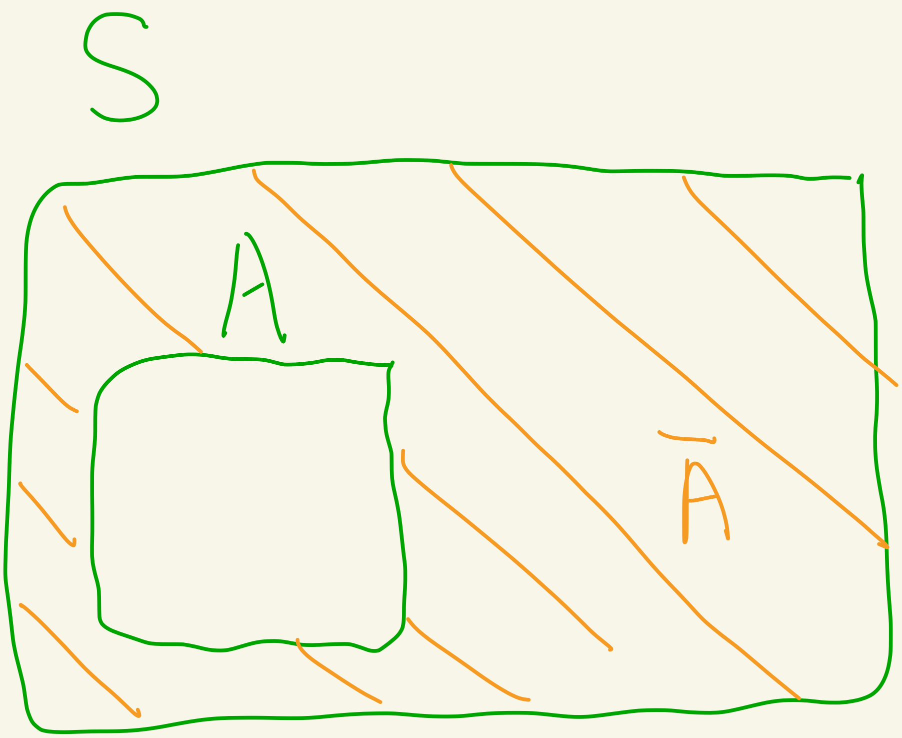
8/28/23

<u>Def</u>: Suppose S is some set and suppose $A \subseteq S$.

The <u>complement</u> of A in S is

$$\overline{A} = \{ x \mid x \in S \text{ and } x \notin A \}$$

read: $\overline{A}$ consists of all x where x is in S and x is not in A



Some other notations for $\overline{A}$ are

$S - A$

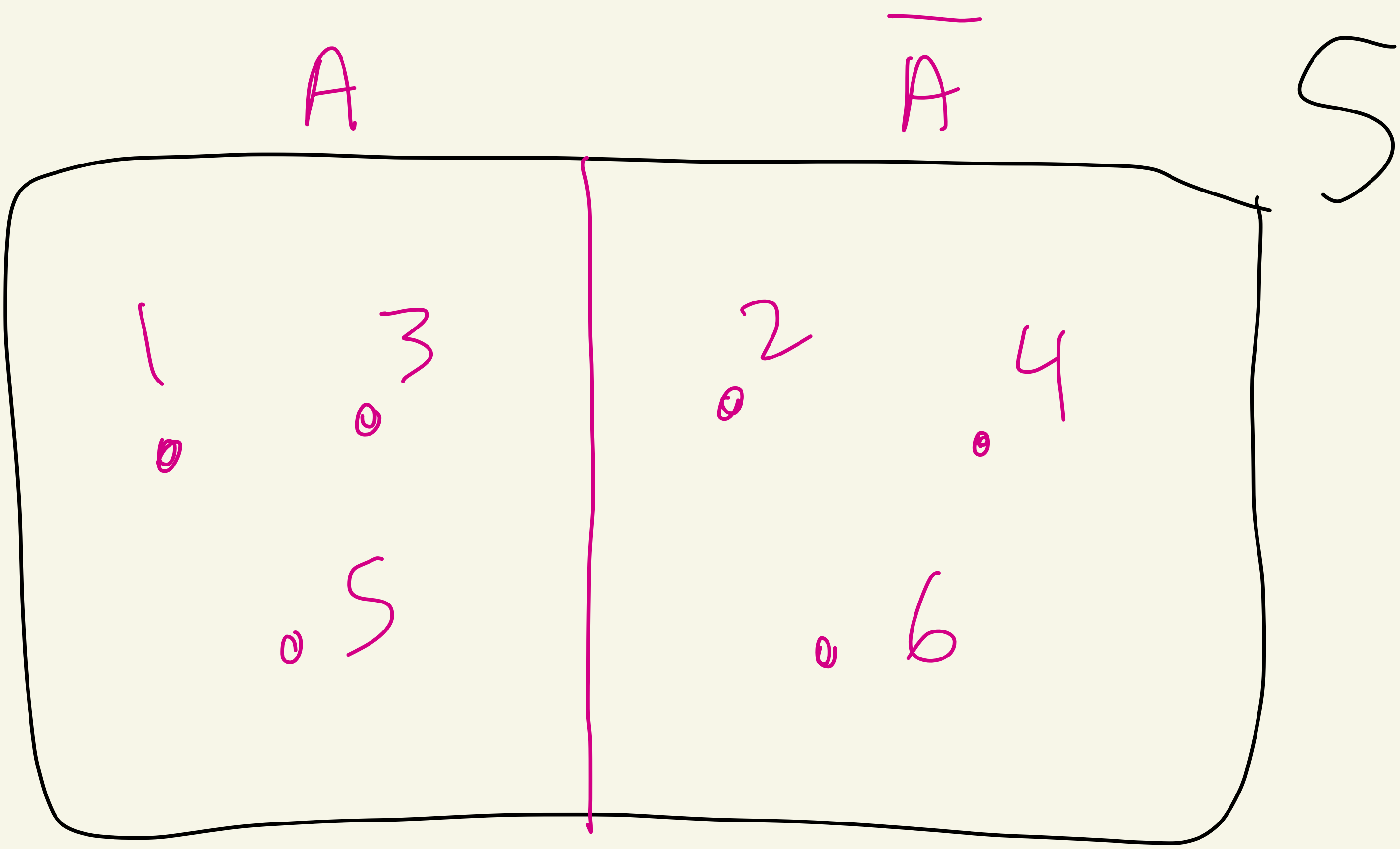
A^c

<u>Ex:</u> $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$ ← odd #s

Then

$\bar{A} = \{2, 4, 6\}$

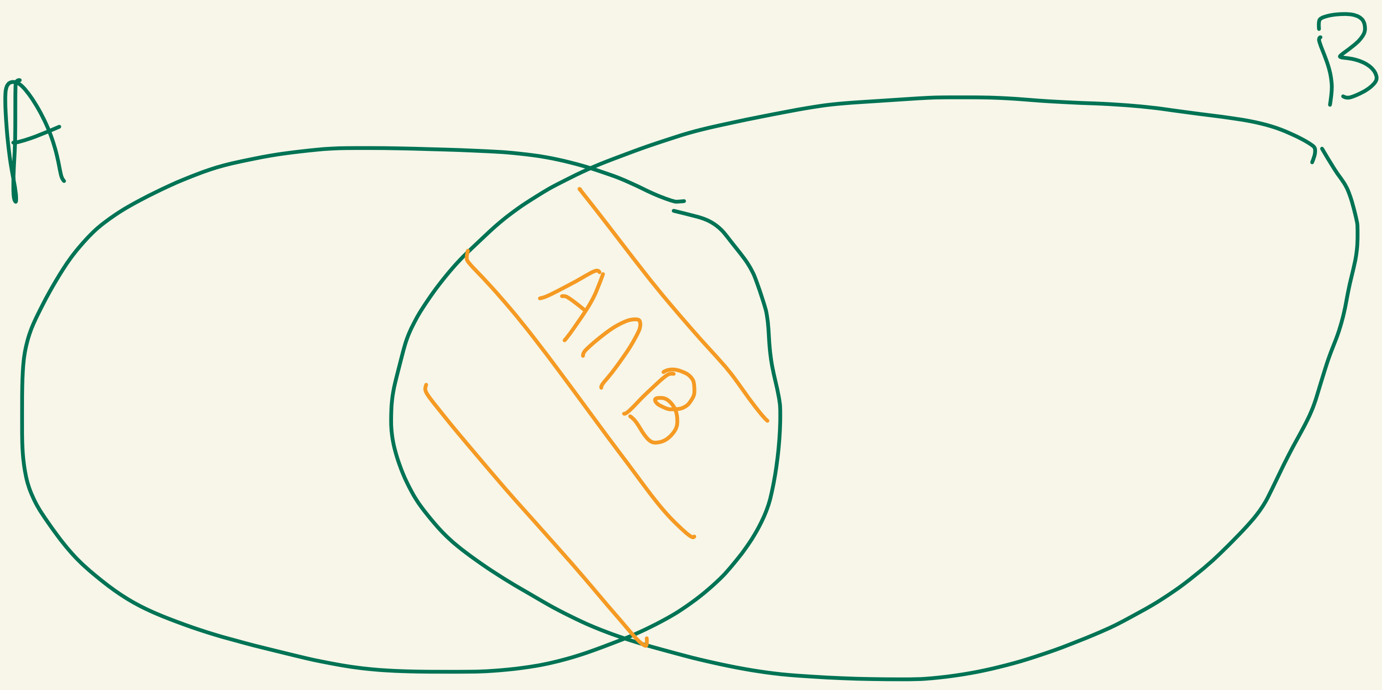


Def: Let A and B be sets.

The intersection of A and B is

$$A \cap B = \underbrace{\{x \mid x \in A \text{ and } x \in B\}}_{}$$

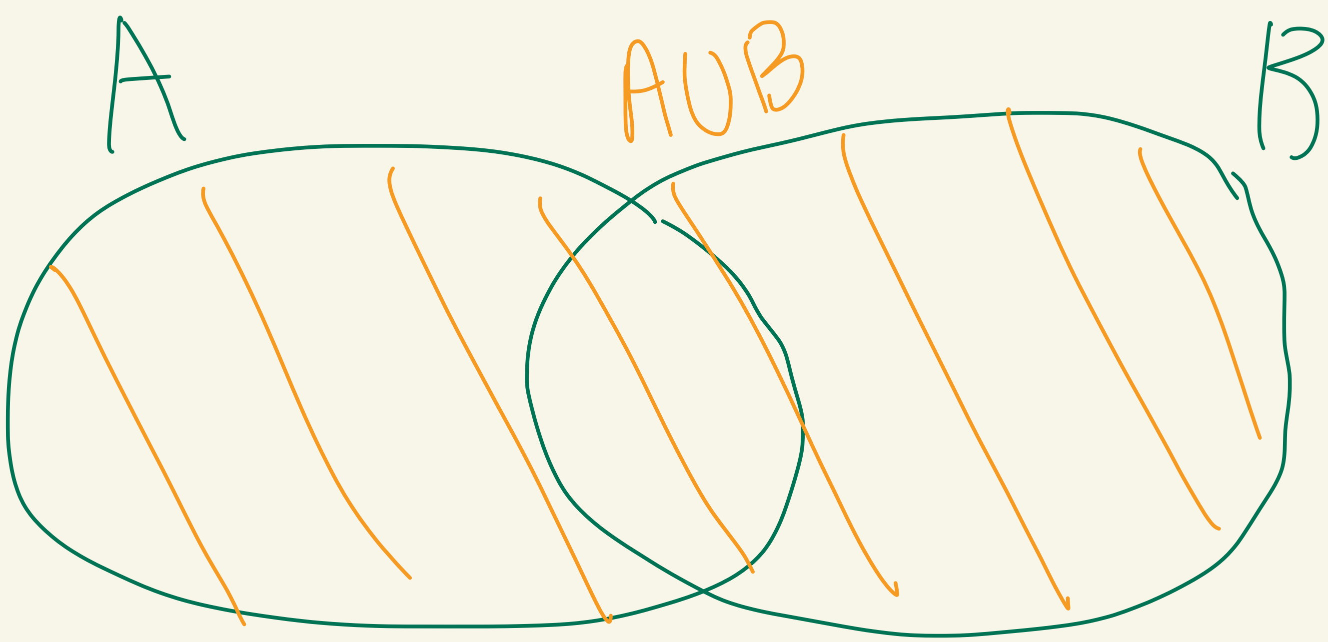
$A \cap B$ consists of all x where x is in A and x is in B



The union of A and B is

$$A \cup B = \{x \mid \underbrace{x \in A \text{ or } x \in B}_{\text{all } x \text{ where } x \text{ is in } A \text{ or } x \text{ is in } B}\}$$

[in math "or" can mean both $x \in A, x \in B$]



<u>Def</u>: The <u>empty set</u> is the set with no elements. It's denoted by

$$\emptyset \quad \text{or} \quad \{\ \}$$

<u>Ex</u>: Let S be the sample space for flipping a coin 3 times in a row.

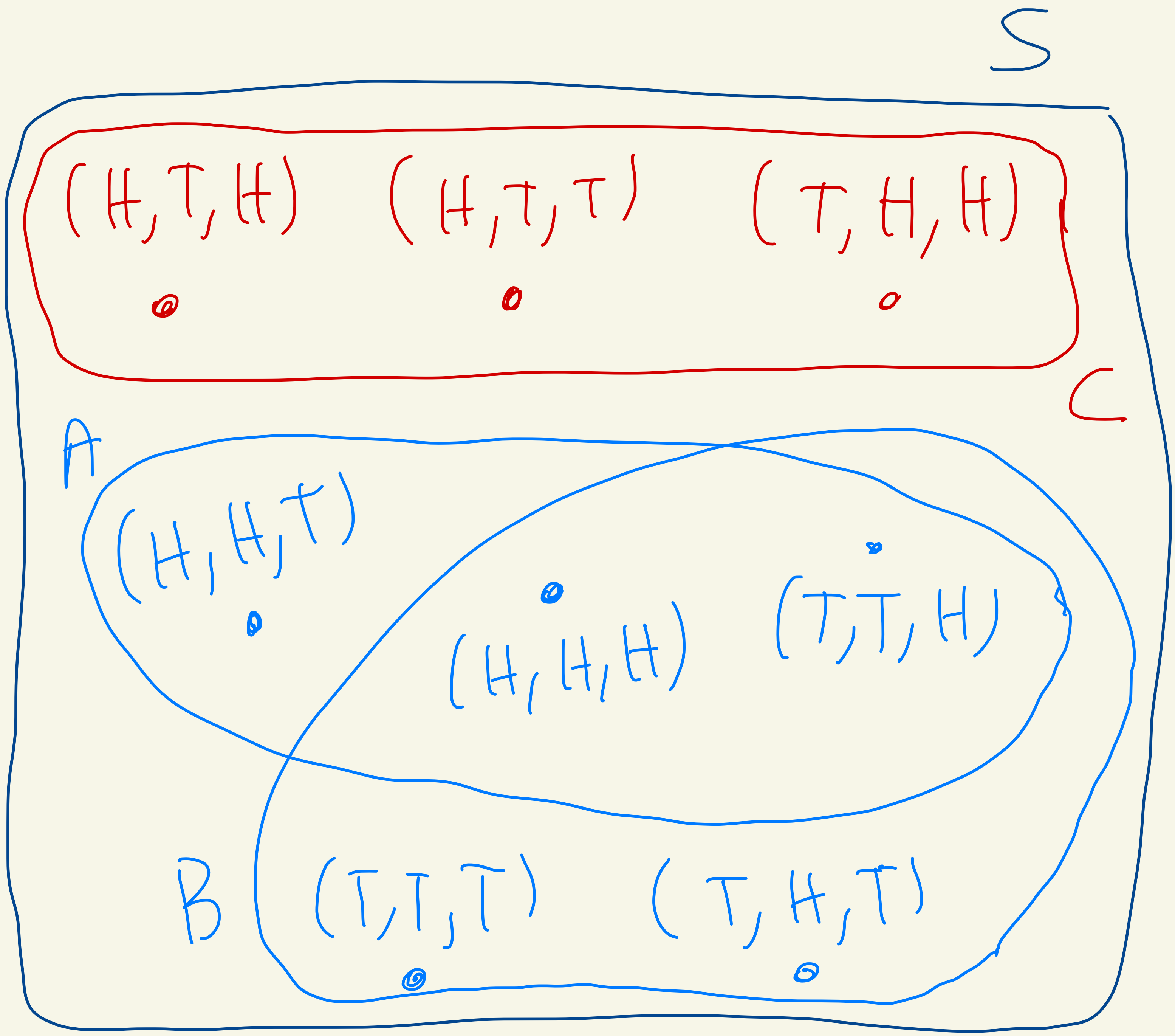
$$S = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)\}$$

Let

$A = \{(H,H,T), (H,H,H), (T,T,H)\}$

$B = \{(T,T,T), (T,T,H), (H,H,H), (T,H,T)\}$

$C = \{(H,T,H), (H,T,T), (T,H,H)\}$



Then,

$$A \cup B = \{(H,H,T), (H,H,H), (T,T,H), (T,T,T), (T,H,T)\}$$

$$A \cap B = \{(H,H,H), (T,T,H)\}$$

$A \cap C = \phi$ $\qquad$ $B \cap C = \phi$

<u>Def:</u> Two sets X and Y are <u>disjoint</u> if $X \cap Y = \phi$.

<u>Ex:</u> In the previous example A and C are disjoint. Also, B and C were disjoint.

<u>Def:</u> Let $A_1, A_2, \ldots, A_n$ be sets. Define

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

$$= \left\{ x \,\middle|\, \begin{array}{l} x \in A_1 \text{ and } x \in A_2 \\ \text{and } \ldots \text{ and } x \in A_n \end{array} \right\}$$

$$= \left\{ x \,\middle|\, x \in A_i \text{ for all } 1 \leq i \leq n \right\}$$

and

$$\bigcup_{i=1}^{n} A_i = \left\{ x \,\middle|\, \begin{array}{l} x \in A_1 \text{ or } x \in A_2 \text{ or} \\ \ldots \text{ or } x \in A_n \end{array} \right\}$$

$$= \left\{ x \,\middle|\, x \in A_i \text{ for some } 1 \leq i \leq n \right\}$$

$$= \left\{ x \,\middle|\, \begin{array}{l} x \text{ is in at least} \\ \text{one of } A_1, A_2, \ldots, A_n \end{array} \right\}$$

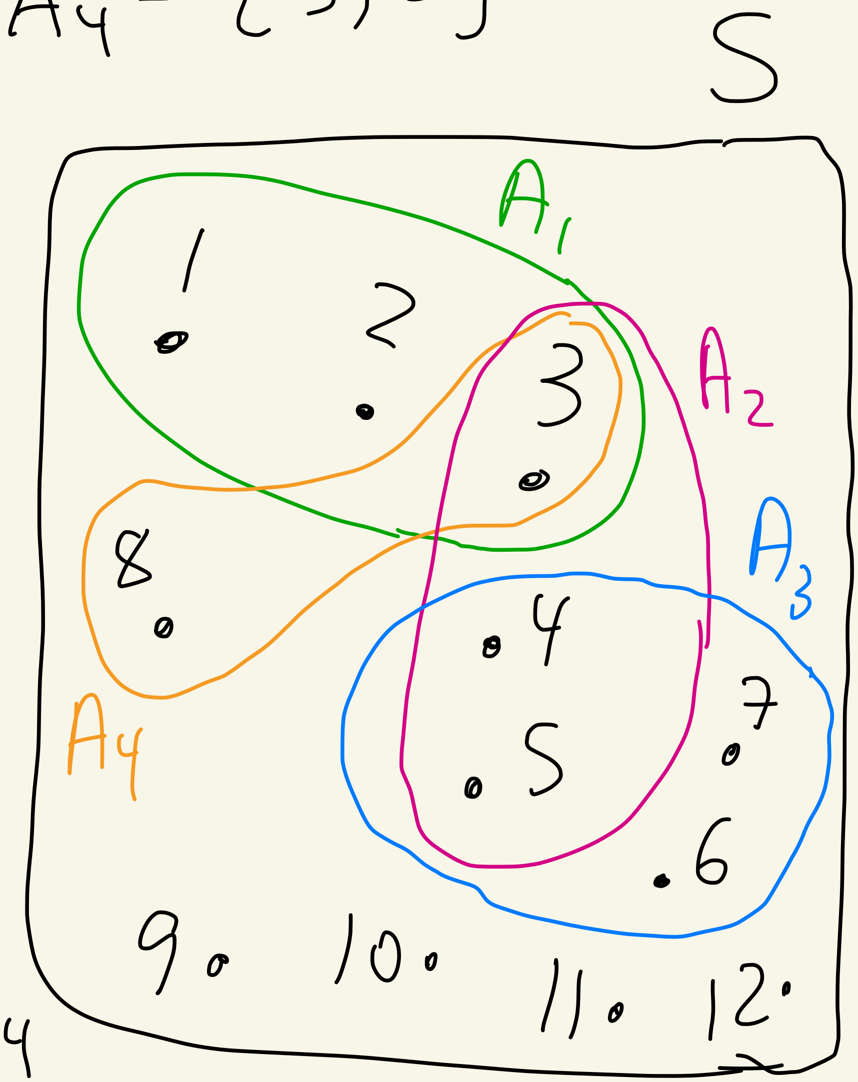
Ex: $S = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

represents rolling a 12-sided die (dodecahedron)

Let

$A_1 = \{1,2,3\}$ $\quad$ $A_3 = \{4,5,6,7\}$

$A_2 = \{3,4,5\}$ $\quad$ $A_4 = \{3,8\}$



$$\bigcup_{i=1}^{4} A_i = A_1 \cup A_2 \cup A_3 \cup A_4$$
$$= \{1,2,3,4,5,6,7,8\}$$

$A_1 \cup A_2 \cup A_4$
$$= \{1,2,3,4,5,8\}$$

$$\bigcap_{i=1}^{4} A_i = A_1 \cap A_2 \cap A_3 \cap A_4$$
$$= \phi$$

$$A_1 \cap A_2 \cap A_4 = \{3\}$$

$\boxed{\text{Def:}}$ Suppose $A_1, A_2, A_3, \ldots$ are an infinite number of sets.

Define

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots$$
$$= \left\{ x \;\middle|\; \begin{array}{l} x \text{ is in every one} \\ \text{of the } A_i \end{array} \right\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots$$
$$= \left\{ x \;\middle|\; \begin{array}{l} x \text{ is in at least} \\ \text{one of the } A_i \end{array} \right\}$$

Ex: The set of integers is

$$\mathbb{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

For each $i \geq 1$, define

$$A_i = \{x \mid x \in \mathbb{Z} \text{ and } -i \leq x \leq i\}$$

Then,

$$A_1 = \{-1, 0, 1\}$$
$$A_2 = \{-2, -1, 0, 1, 2\}$$
$$A_3 = \{-3, -2, -1, 0, 1, 2, 3\}$$
$$A_4 = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$
$$A_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$
$$\vdots$$

Then,

$$\bigcap_{i=1}^{\infty} A_i = \{-1, 0, 1\}$$

A_1

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}$$

all the integers

Def: Let A and B be sets. The Cartesian product of A and B is

$$\underbrace{A \times B}_{\text{read ``A cross B''}} = \underbrace{\{(a,b) \mid a \in A \text{ and } b \in B\}}_{\text{all elements of the form } (a,b) \text{ where}}$$

$a \in A$ and $b \in B$

Ex: Let $A = \{H, T\}$
and $B = \{1, 2, 3, 4\}$

Then,

$$A \times B = \{(H,1), (H,2), (H,3), (H,4), (T,1), (T,2), (T,3), (T,4)\}$$

$$A \times A = \{(H,H), (H,T), (T,T), (T,H)\}$$

$$B \times A = \{(1,H), (2,H), (3,H), (4,H), (1,T), (2,T), (3,T), (4,T)\}$$

Def: Let A and B be sets. A function f from A to B, notated $f: A \to B$, is a rule that assigns to each element of A a distict element of B.