Math 4740

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Topic 1 - Sets and Probability Spaces

Def: A set is a collection of objects/elements.
If $x$ is an element of a set $S$ then we write $x \in S$.
"x is in $S$ " Set $S$, then we write $x \notin S$

$$
\underbrace{x \in S}_{\begin{array}{c}
\text { read: } \\
\text { "x is not } \\
\text { in } S^{\prime \prime}
\end{array}}
$$

If $S$ has a finite number of elements then the size of $S$ is denoted by $|S|$

Ex: Let's make a set that models rolling a 6 -sided die.

$$
\begin{aligned}
& \text { Let } \\
& S=\{1,2,3,4,5,6\}
\end{aligned}
$$

We have that:
OO ut comes rolling a 6 -sided die Later we will call this the sample space

Note: In a set, order doesn't matter.
So,

$$
\{1,2,3,4,5,6\}=\{2,3,1,6,5,4\}
$$

Note: Set's can't have duplicates. $\{1,1,5\}$ is not a set.

General way to make a set

$$
\left\{\begin{array}{l|l}
\text { description of } \\
\text { what the } \\
\text { elements in } \\
\text { the set look } & \begin{array}{l}
\text { conditions } \\
\text { that the } \\
\text { elements } \\
\text { must satisfy } \\
\text { to be in the }
\end{array}
\end{array}\right\}
$$

Some people use : instead of

Ex: Let's model rolling two G-sided die, one green and one red.

$$
\left.\begin{array}{rl}
S= & \{\left.\underbrace{(9, r)}_{\begin{array}{c}
\text { ordered pair. Use when } \\
\text { order mattes }
\end{array}} \right\rvert\, \begin{array}{l}
g=1,2,3,4,5,6 \\
r=1,2,3,4,5,6
\end{array}\}
\end{array}\right\}
$$

$$
\begin{aligned}
& (3,5) \leftarrow \text { means green die is } 3 \\
& \text { red die is } 5
\end{aligned}
$$

$$
\text { red die is } 5
$$

Note: $|S|=36$

Def: Let $A$ and $B$ be sets. We say that $A$ is a subset of $B$, and write $A \subseteq B$, if every element of $A$ is also an element of $B$.


Ex: Consider rolling a 6-sided
 We will say that E "occured" if when we roll the die we get either 1,3, or 5 .

Ex: Suppose we roll two 6 -side die, one green and one red.

$$
S=\left\{(g, r) \left\lvert\, \begin{array}{l}
g=1,2,3,4,5,6 \\
r=1,2,3,4,5,6
\end{array}\right.\right\}
$$

Let's make a subset of $S$ containing the rolls where the dice add up to 7 .

$$
\begin{array}{r}
E=\{(1,6),(2,5),(3,4),(4,3) \\
(5,2),(6,1)\}
\end{array}
$$

Here $E \subseteq S$.
Later we will say that $E$ is the event that the dice add up to 7

Ex: Suppose we flip a coin three times in a row and record each tine we get $H=$ heads or $T=$ tails.

Sample space:
vi parentheses to mean order matters

$$
\left.\begin{array}{rl}
S=\{ & (H, H, H)^{l}, \\
(H, H, T), \\
& (H, T, H), \\
& (H, T, T), \\
& (T, H, H), \\
& (T, H, T) \\
& (T, T, H),
\end{array}(T, T, T)\right\}
$$

Here (H,T,H) means:
H on 1 st flip
$T$ on Ind flip
H on 3 rd flip

The event E representing exactly one head occurs in three flips would be

$$
E=\{(H, T, T) g(T, H, T),(T, T, H)\}
$$

Note $|S|=8$
and $|E|=3$

