Math 4740

10/4/23

$\boxed{\text{HW 2}}$

$\boxed{5(c)}$ Sequence of length 8 how many have exactly three 0's, three 1's, and two 2's ?

Ex: 0 1 1 0 2 0 1 2

$\boxed{\text{Answer}}$:

| | |
|---|---|
| Step 1: Pick where the three 0's go. # ways = $\binom{8}{3} = \frac{8!}{3!\,5!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{6}\cdot \cancel{5!}} = 56$ | 0 _ 0 _ _ 0 _ _ |
| Step 2: Pick where the three 1's go. | 0 1 0 _ 1 0 1 _ |

$$\binom{5}{3} = \frac{5!}{3!\,2!} = 10$$

<u>Step 3:</u> Pick where the two 2's go.

$$\binom{2}{2} = 1$$

$\underline{0}\ \underline{1}\ \underline{0}\ \underline{2}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{2}$

$$\text{Answer} = 56 \cdot 10 \cdot 1 = \boxed{560}$$

Hw 2

(13) A coin is tossed 20 times.

(a) What is the probability that at least 2 heads occurs?

(b) what is the probability that at most 3 heads occur?

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Solution:

Sample space size $= 2^{20}$

$= 1{,}048{,}576$

(a) Want: $P\left(\geq 2 \text{ heads occuring}\right)$

$= 1 - P\left(< 2 \text{ heads occuring}\right)$

$= 1 - P\left(\begin{matrix}\text{no heads} \\ \text{occuring}\end{matrix}\right) - P\left(\begin{matrix}\text{exactly 1} \\ \text{head occuring}\end{matrix}\right)$

$$= 1 - \frac{1}{2^{20}} - \frac{\binom{20}{1}}{2^{20}}$$

$\underline{T}\ \underline{T}\ \underline{T} \ldots\ldots \underline{T}$

all tails

$\underline{T}\ \underline{H}\ \underline{T}\ \underline{T} \ldots \underline{T}$

20 choices where the head can go and the rest are tails

$\binom{20}{1} = 20$ of these

$$= 1 - \frac{1}{2^{20}} - \frac{20}{2^{20}}$$

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(b) Want:

$P(\leq 3 \text{ heads occuring})$

$= P\left(\begin{matrix}\text{no heads}\\ \text{occuring}\end{matrix}\right) + P\left(\begin{matrix}\text{exactly 1}\\ \text{head occuring}\end{matrix}\right)$
$+ P\left(\begin{matrix}\text{exactly 2}\\ \text{heads occuring}\end{matrix}\right) + P\left(\begin{matrix}\text{exactly 3}\\ \text{heads occuring}\end{matrix}\right)$

$$= \frac{1}{2^{20}} + \frac{20}{2^{20}} + \frac{\binom{20}{2}}{2^{20}} + \frac{\binom{20}{3}}{2^{20}}$$

T T T T T H T T T T T T T H T T T T T T

$\binom{20}{2}$ ← pick where heads go

$\binom{18}{18} = 1$ ← # ways to now put tails in

$$= \frac{1}{2^{20}} + \frac{20}{2^{20}} + \frac{\frac{20\cdot 19}{2}}{2^{20}} + \frac{\frac{20\cdot 19\cdot 18}{6}}{2^{20}}$$

$$= \frac{1}{2^{20}} + \frac{20}{2^{20}} + \frac{190}{2^{20}} + \frac{1140}{2^{20}}$$

$\boxed{\text{HW 2}}$

(11) Roll ten 6-sided die.

Probability exactly one 4, exactly six 5's, the other three rolls are anything other than 4's and 5's.

Ex: 5 1 5 5 1 5 4 5 5 3

Sample space size $= 6^{10}$

$= 60,466,176$

\# of sequences we want to count is:

| | |
|---|---|
| Step 1: Put where 4 goes. $\binom{10}{1} = 10$ | _ _ _ _ _ _ 4 _ _ _ |

Step 2: Now put the six 5's in.

$$\binom{9}{6} = \frac{9!}{6!\,3!} = \frac{9\cdot 8\cdot 7}{6} = 84$$

5 _ 5 5 _ 5 4 5 5 _

Step 3: The remaining three spots can be 1, 2, 3, or 6.

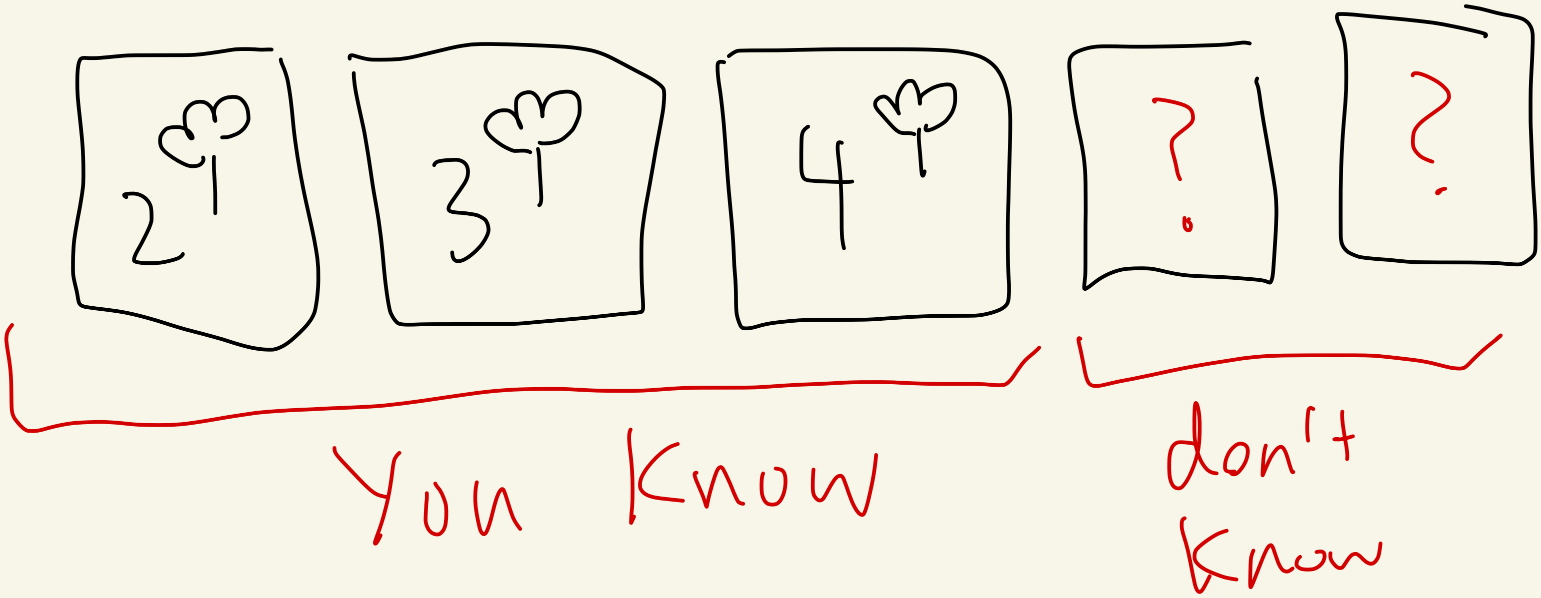
$$\#\text{ ways} = 4\cdot 4\cdot 4 = 64$$

5 1 5 5 1 5 4 5 5 3

4 choices 4 choices 4 choices

$$\text{Answer} = \frac{10\cdot 84\cdot 64}{6^{10}} = \frac{53,760}{60,466,17}$$

HW 2 16 Dealt 5 cards from 52-card deck




$$\text{sample space size} = \binom{52-3}{2} = \binom{49}{2}$$

$$= \frac{49!}{2! \cdot 47!} = \frac{49 \cdot 48}{2} = \boxed{1{,}176}$$

(a) What's the chances the other two cards are clubs so you have a flush?

$$\binom{\overbrace{13}^{\#\text{ clubs}} - \overbrace{3}^{\text{remove } 2♣, 3♣, 4♣}}{\underbrace{2}_{\text{pick } 2}} = \binom{10}{2} = \frac{10!}{2!8!} = \frac{10 \cdot 9}{2}$$

$$= 45$$

$$\text{answer} = \frac{45}{1{,}176} \approx 0.03826$$

$$\boxed{\frac{10 \cdot 9}{49 \cdot 48}}$$

other way

order matters

(b) odds you get straight but not straight flush?

Straights

\# of these

A? 2♣ 3♣ 4♣ 5? ← $4 \cdot 4 = 16$

2♣ 3♣ 4♣ 5? 6? ← $4 \cdot 4 = 16$

Total # straights $=$ $\boxed{16+16 = 32}$

Straight flushes

A♣ 2♣ 3♣ 4♣ 5♣

2♣ 3♣ 4♣ 5♣ 6♣

2 of these

$$\text{Answer} = \frac{32 - 2}{1,176} = \frac{30}{1,176}$$