$$
\begin{gathered}
\text { Math } 4740 \\
10 / 23 / 23
\end{gathered}
$$

Let's analyze some
Roulette bets
Sample space for American wheel

$$
\begin{array}{r}
S=\left\{\begin{array}{l}
0,00,1,2,3,4,5,6,7,8 \\
9,10,11,12,13,14,15,16, \\
17,18,19,20,21,22,23 \\
\\
24,25,26,27,28,24,30, \\
\\
31,32,33,34,35,36\}
\end{array}, \$\right. \text {, }
\end{array}
$$

Each number is equally likely with probability $\frac{1}{38}$.

Straight up bet (35:1 payout)
Suppose we bet $\$ 1$ on 27 .
Let $\mathbb{E}$ be the amount won or lost.

$$
X(w)= \begin{cases}-1 & \text { if } w \neq 27 \\ 35 & \text { if } w=27\end{cases}
$$

Then,

$$
\begin{aligned}
& \text { (hen) } \begin{aligned}
E[\bar{x}] & =(-1) \cdot P(\bar{x}=-1) \\
& +(35) \cdot P(\bar{x}=35) \\
& =(-1)\left(\frac{37}{38}\right)+(35)\left(\frac{1}{38}\right) \\
& =\frac{-2}{38} \approx-0.0526
\end{aligned}
\end{aligned}
$$

So on average we lose $5.26 \nmid$ Per dollar bet

The casino pays 35:1 on a straight up bet.
What are the real odds (ie the odds against)?
Here the event we are betting on is $E=\{27\}$.
odds against $E=\frac{P(\bar{E})}{P(E)}=\frac{37 / 38}{1 / 38}$

$$
=\frac{37}{1}
$$

The odds against are $37: 1$.
What's the expected value if the casino paid $37: 1$ ?

The new random variable is:

$$
Z(w)= \begin{cases}-1 & \text { if } w \neq 27 \\ 37 & \text { if } w=27\end{cases}
$$

Then,

$$
\begin{aligned}
E[\mathcal{I}] & = ( - 1 ) \sqrt { ( \frac { 3 7 } { 3 8 } ) } + ( 3 7 ) \longdiv { ( \frac { 1 } { 3 8 } ) } \\
& =0
\end{aligned}
$$

So if the casino paid 37:1 on a straight up bet everyone breaks even in the long run.

Column bet ( $2: 1$ payout)
Suppose we bet $\$ 1$ on the third column. So we ace betting on the event

$$
\begin{gathered}
E=\{3,6,9,12,15,18,21,24,27, \\
30,33,36\} .
\end{gathered}
$$

Let X be the amount won or Then,

$$
X(w)=\left\{\begin{array}{ll}
-1 & \text { if } w \notin E \\
2 & \text { if } w \in E
\end{array} \begin{array}{l}
w \notin E \\
\begin{array}{l}
\text { Then, } \\
\text { means } \\
w \text { is not } \\
\text { in } E \\
w \in E \\
\text { means }
\end{array}
\end{array}\right.
$$

$$
\text { that } w \text { is in } E
$$

Then,

$$
\begin{aligned}
E[\bar{X}] & =(-1) \cdot P(X=-1) \\
& +(2) \cdot P(X=2) \\
& =(-1)\left(\frac{26}{38}\right)+(2)\left(\frac{12}{38}\right) \\
& =\frac{-2}{38} \approx-0.0526
\end{aligned}
$$

So on average we lose 5,264 per dellur bet in the long run.

What are the true odds for $E$ (ie the odds against $E$ )?
It is $\frac{P(E)}{P(E)}=\frac{26 / 38}{12 / 38}=\frac{26}{12}=\frac{13}{6}$
Ie 13:6
If the casino paid 13:6 on column bets then the expected value would be:

$$
(-1)\left(\frac{26}{38}\right)+\left(\frac{13}{6}\right)\left(\frac{12}{38}\right)=0
$$

Then everyone breaks even in the long run.


Craps
The main bet in craps is called the pass line bet.
People place their bets on the table and the game starts.
Suppose we put money on the pass line.


Some player (called the shooter) rolls the dice.
Two 6-sided dice ace rolled.
The first roll is called "the come out roll."

The sum of the dice is measured on each roll.
$\left.\begin{array}{l}\text { case 1: If a } 7 \text { or } 11 \\ \text { is rolled, then we win } \\ \text { the pass line bet. }\end{array}\right] \begin{aligned} & \text { this } \\ & \text { roll } \\ & \text { is } \\ & \text { called } \\ & \text { naturall }\end{aligned}$
case 2: If a 2,3 , or 12 is rolled, then we lose the pass line bet.

Case 3: Suppose a 4, 5, 6, 8,9, or 10 is rolled. The number rolled is called the "point." Now the dice are rolled over and over again until either 7 is rolled or the point is rolled again.
If 7 comes vp first, then we lose the pass line bet.
If the point comer up first then we win the pass line bet.

- After one of the 3 cases a buve occurs, then the game starts over with a new come out roll.
- The casino plays 1:1 on a pass line bet.

Let's see some example games
pass line
bet $\$ 2$

| come out <br> roll |  |
| :---: | :---: |
| $\because \square$ | $\ddots$ |

7 is rolled un come out roll game over we Won $\$ 2$

$$
\begin{gathered}
\text { pass line } \\
\text { bet }
\end{gathered} \$ 50
$$

| come out |
| :---: |
| roll |
| $0 \square \square$ |

3 is rolled un come out coll game over we lost $\$ 50$

$$
\begin{array}{|c|c|}
\hline \begin{array}{c}
\text { pars line } \\
\text { bet }
\end{array} & \$ 100 \\
\hline
\end{array}
$$



