Math 4740 10/23/23

Let's analyze some
Roulette bets
Sample space for American wheel
$$S = \{ 0, 00, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 24, 30, 31, 32, 33, 34, 35, 36 \}$$

Each number is equally likely with probability $\frac{1}{38}$.

Straight up bet (35:1 payout) Suppose we bet \$1 on 27. Let X be the amount won or lost. $X(w) = \begin{cases} -1 & \text{if } w \neq 27 \\ 35 & \text{if } w = 77 \end{cases}$. if w=27 Then, $E[X] = (-1) \cdot P(X = -1)$ $+(35)\cdot P(X=35)$ $= \left(-1\right)\left(\frac{37}{38}\right) + \left(35\right)\left(\frac{1}{38}\right)$ $=\frac{-2}{-38} \approx -0.0526$ So on average we lose 5.26¢ Per Jollar bet

35:1 01 The casino pays bet. a straight up real odds What are the (ie the odds against)? Here the event we are betting on is $E = \{2,7\}$. P(E) 37/38 against $E = \frac{P(E)}{P(E)} = \frac{37/38}{738}$ odds $= \frac{37}{1}$ The odds against are 37:1. What's the expected value if the casino paid 37:1?

The new random variable is: $\sum (w) = \begin{cases} -1 & \text{if } w \neq 27 \\ 37 & \text{if } w = 27 \end{cases}$ Then, $P(\underline{X}=-1)$ $p(\underline{X}=37)$ $E[\underline{X}] = (-1)(\frac{37}{38}) + (37)(\frac{1}{38})$ = 0So if the casino paid 37:1 on a straight up bet everyone breaks even in the long run.

Column bet (2°.1 payout) Suppose we bet \$1 on the third column. So we are betting on the event $E = \{23, 6, 9, 12, 15, 18, 21, 24, 27\}$ 30,33,36 f. Let X be the amount won or Jost. Then, if w & E W∉E $\begin{bmatrix} -1 \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} -1 \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}$ means w is not if we E INE WEE means that wis in E

Then,

$$E[X] = (-1) \cdot P(X=-1) + (2) \cdot P(X=2) + (2) \cdot P(X=2) = (-1)(\frac{26}{38}) + (2)(\frac{12}{38}) = \frac{-2}{38} \approx -0.0526$$

What are the true udds for E (ie the odds against E)? It is $\frac{P(E)}{P(E)} = \frac{26/38}{12/38} = \frac{26}{12} = \frac{13}{6}$ Ie 13:6 If the casino paid 13:6 on column bets then the expected value would be: $\binom{-1}{38} + \binom{13}{6} \binom{12}{38} = 0$ breaks even Then everyone In the long run.



Craps

The main bet in craps is called the pass line bet. People place their bets on the table and the game starts. Suppore we put money on the pass line. Pusc Inne Pass SWY)