Math 4740

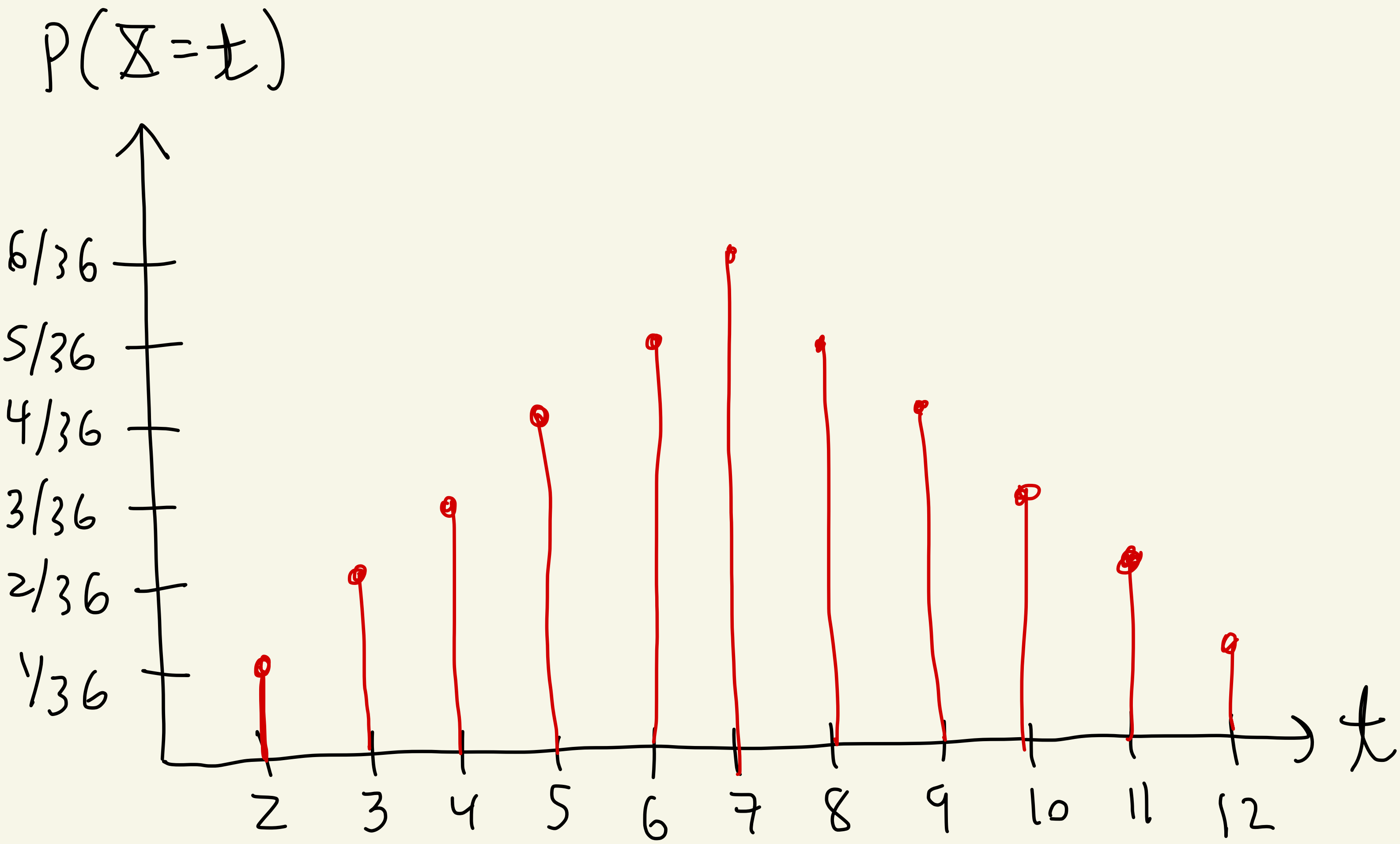
10/18/23

(Topic 4 continued...)

Ex continued...

roll two 6-sided dice

X = sum of the dice



Then,

$$E[X] = \sum_{t=2}^{12} t \cdot P(X = t)$$

$$= (2)\left(\frac{1}{36}\right) + (3)\left(\frac{2}{36}\right) + (4)\left(\frac{3}{36}\right)$$

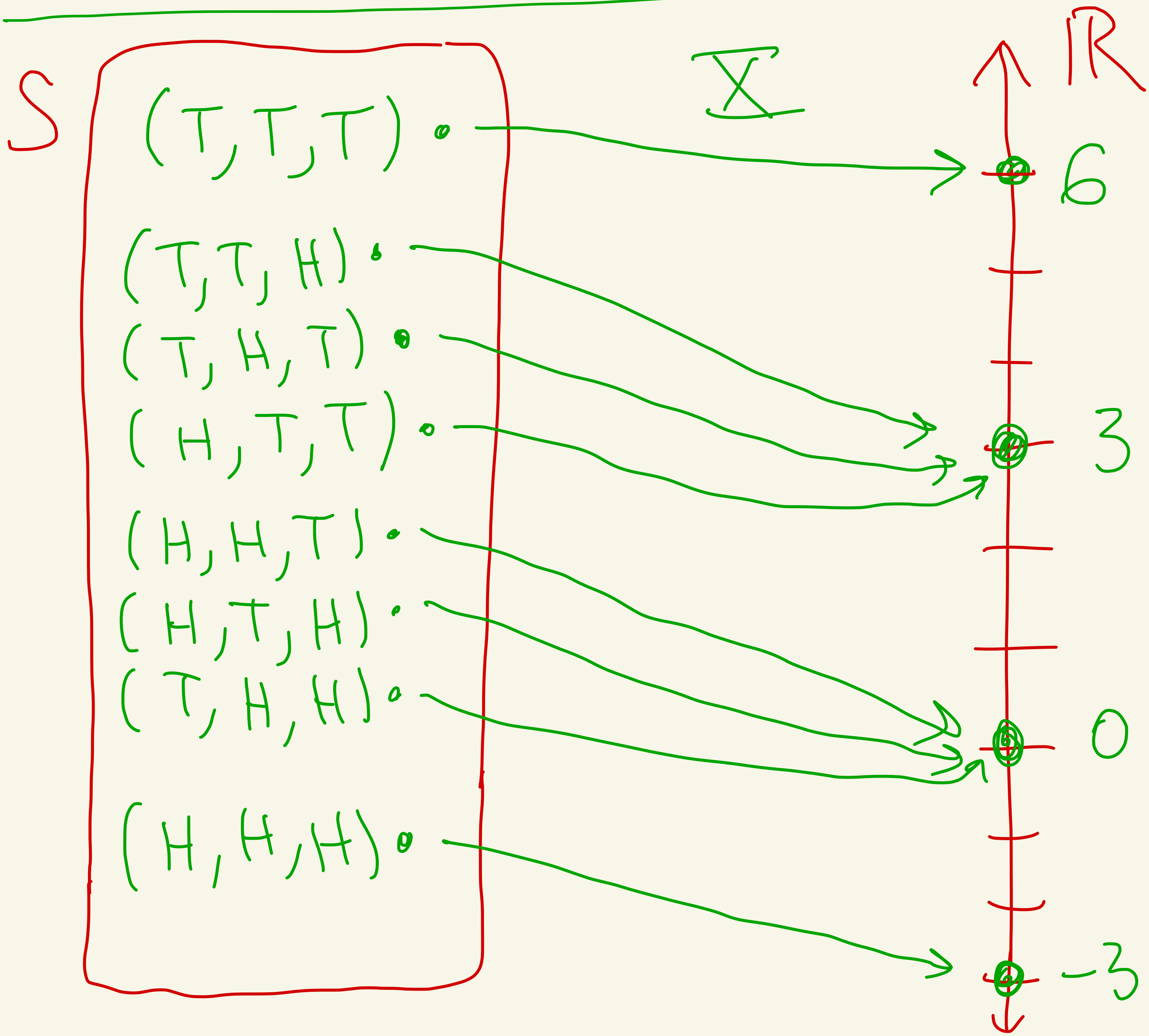
$$+ (5)\left(\frac{4}{36}\right) + (6)\left(\frac{5}{36}\right) + (7)\left(\frac{6}{36}\right)$$

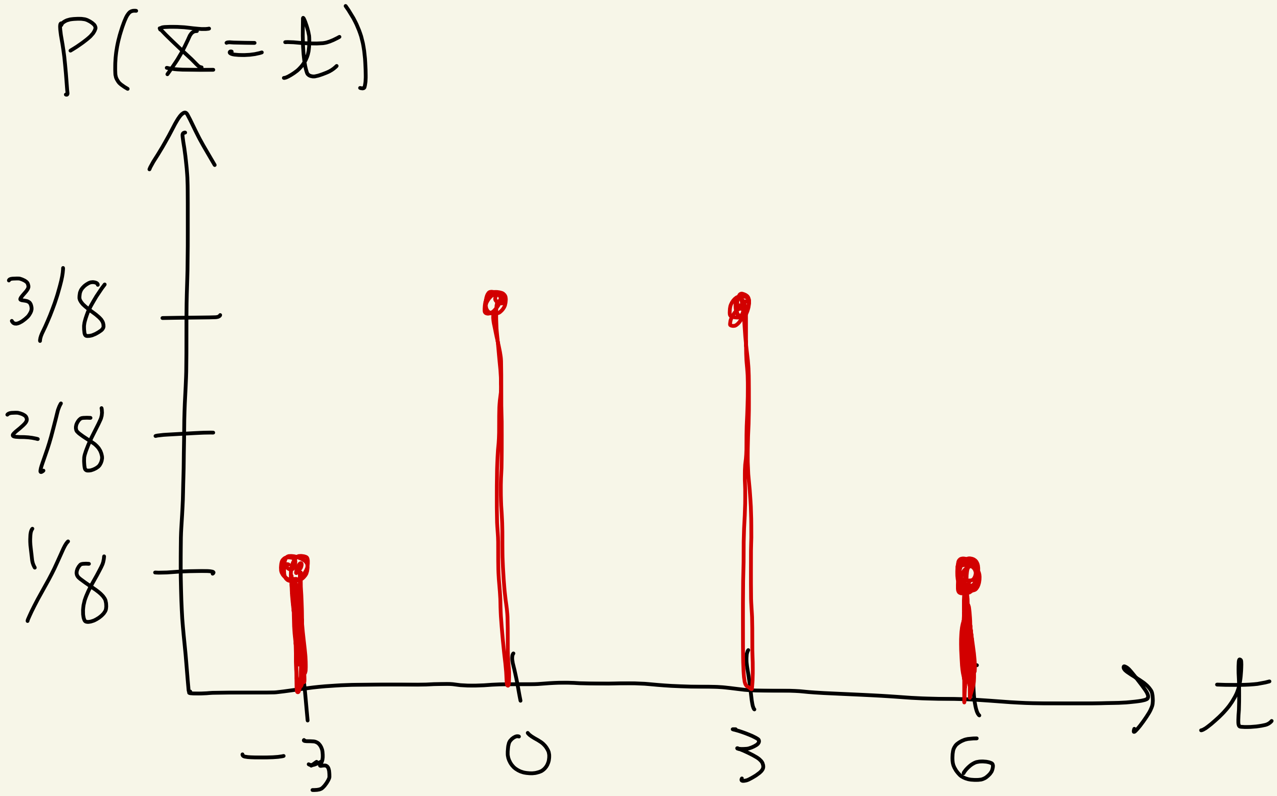
$$+ (8)\left(\frac{5}{36}\right) + (9)\left(\frac{4}{36}\right) + (10)\left(\frac{3}{36}\right)$$

$$+ (11)\left(\frac{2}{36}\right) + (12)\left(\frac{1}{36}\right)$$

$$= 7$$

Ex: Suppose you flip a coin 3 times. For every head you lose \$1. For every tail you win \$2. Let X be the amount won/lost. Draw X and $p(t) = P(X = t)$. Calculate $E[X]$





$$E[X] = (-3)\left(\frac{1}{8}\right) + (0)\left(\frac{3}{8}\right) + (3)\left(\frac{3}{8}\right) + (6)\left(\frac{1}{8}\right)$$

$$= \frac{-3+9+6}{8} = \frac{12}{8} = 1.5$$

This is saying that if you played the game alot of times on average you'd win \$1.50 per play.

So say you played the game 1 million times then you'd expect to win around

$$(1{,}000{,}000)\cdot(\$1.50)$$

$$= \$1{,}500{,}000.$$

$\boxed{\text{Odds}}$ Let E be an event.

We define

$$\text{odds for } E = \frac{P(E)}{P(\bar{E})} = \frac{P(E)}{1-P(E)}$$

$$\text{odds against } E = \frac{P(\bar{E})}{P(E)} = \frac{1-P(E)}{P(E)}$$

casino uses this

Ex: Suppose we roll a 4-sided die. Let E be the event that we roll a 1.

So, $P(E) = \frac{1}{4}$.

$$\text{Odds for } E = \frac{P(E)}{1-P(E)} = \frac{1/4}{3/4} = \frac{1}{3}$$

written 1:3 read "1 to 3"

$$\text{odds against } E = \frac{1-P(E)}{P(E)} = \frac{3/4}{1/4} = \frac{3}{1}$$

written 3:1 read "3 to 1"

How to convert odds to probabilities

odds for E
$a:b$ $\longrightarrow$ $P(E) = \dfrac{a}{a+b}$

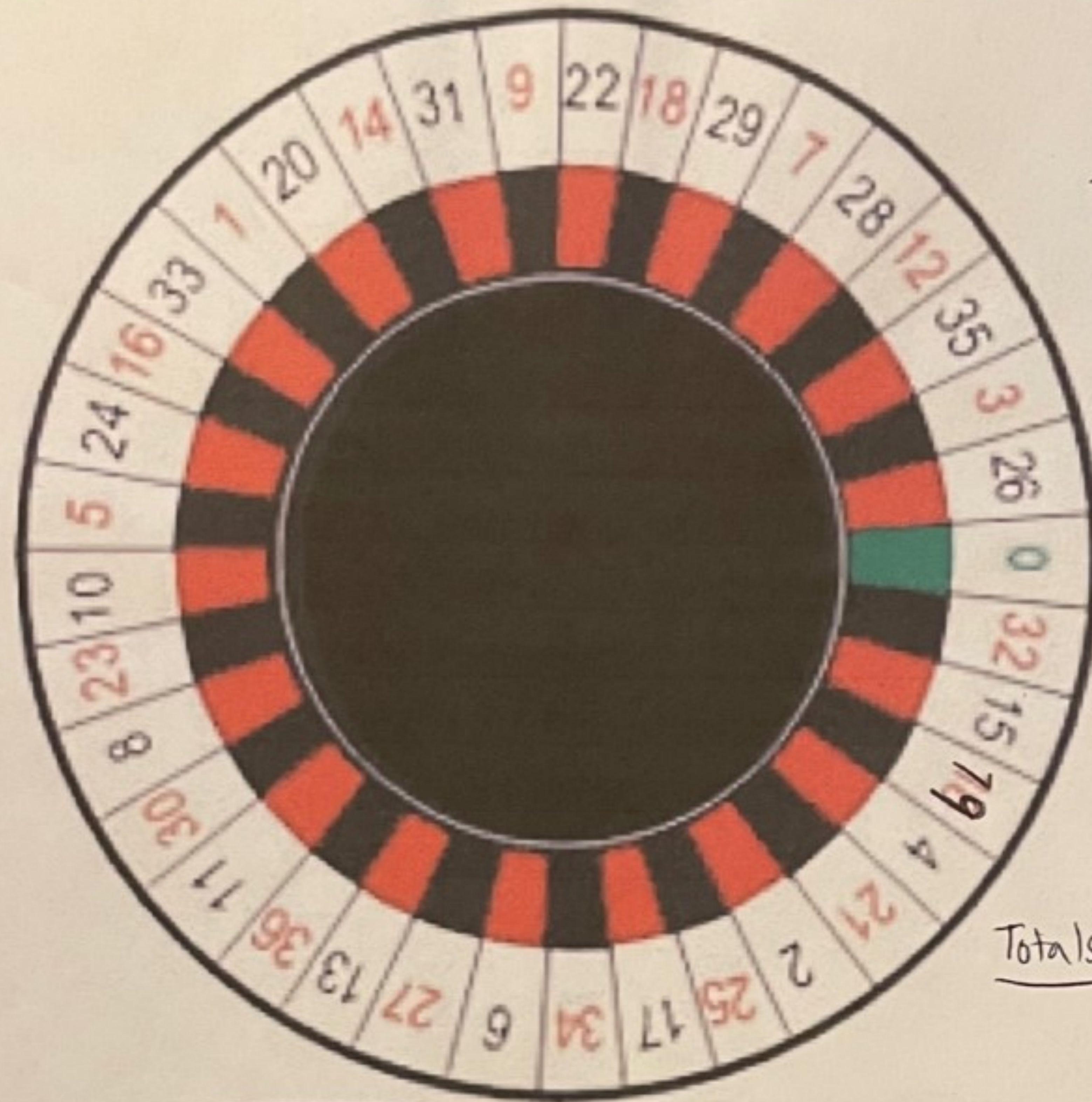
odds against E
$c:d$ $\longrightarrow$ $P(E) = \dfrac{d}{c+d}$

Ex: Suppose the odds for E are $3:5$. Then $P(E) = \dfrac{3}{3+5} = \dfrac{3}{8}$

Ex: Suppose the odds against E are $4:6$. Then $P(E) = \dfrac{6}{4+6} = \dfrac{6}{10}$

Let's learn about Roulette.

EUROPEAN



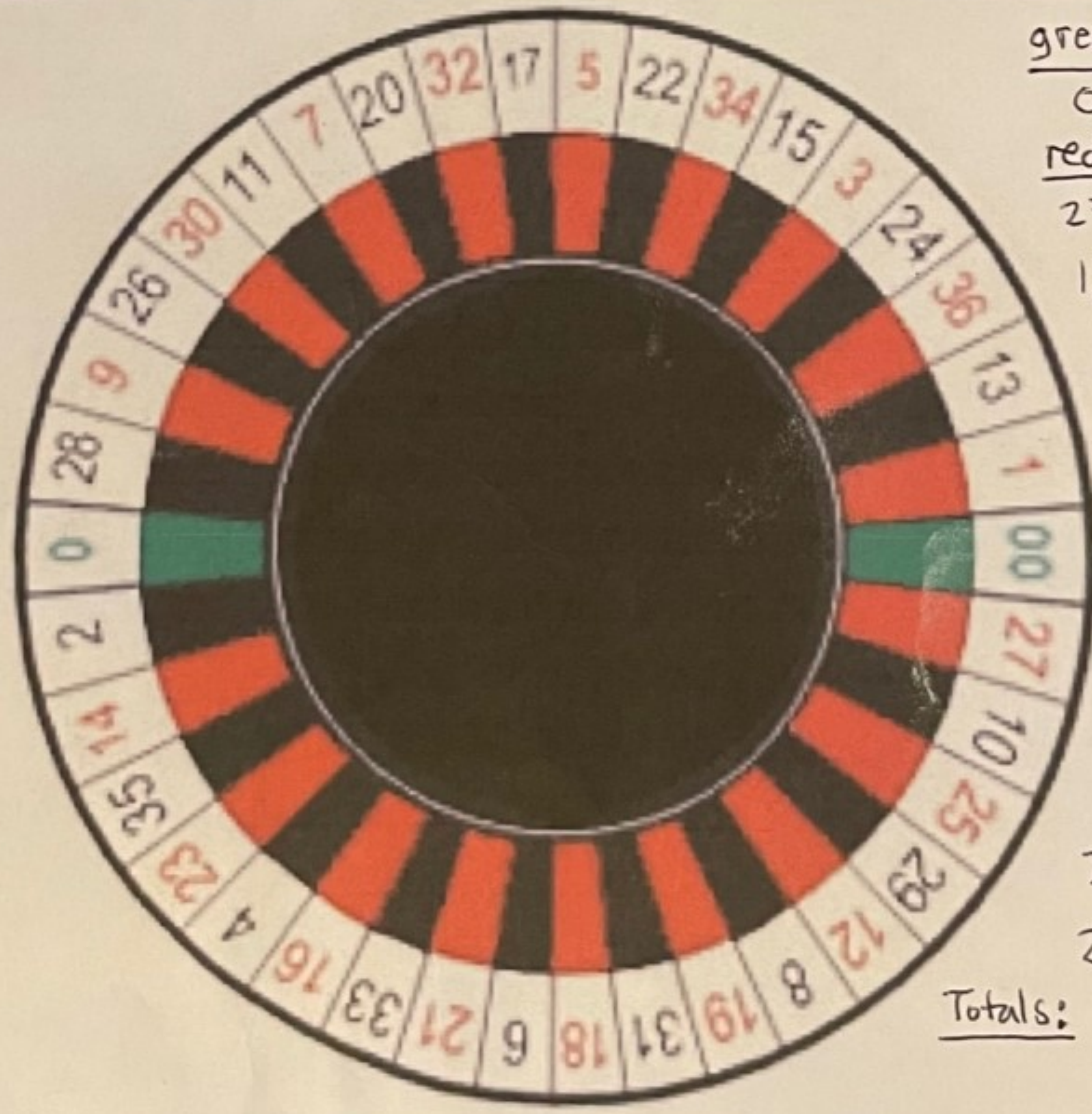
green
0

red
32, 19, 21, 25,
34, 27, 36, 30,
23, 5, 16, 1,
14, 9, 18, 7
12, 3

black
15, 4, 2, 17
6, 13, 11, 8
10, 24, 33, 20
31, 22, 29, 28
35, 26

Totals: 1 green
18 red
18 black
= 37 total

AMERICAN



green
0, 00

red
27, 25, 12, 19
18, 21, 16, 23
14, 9, 30, 7
32, 5, 34, 3
36, 1

black
10, 29, 8, 31
6, 33, 4, 35
2, 28, 26, 11
20, 17, 22, 15
24, 13

Totals: 2 green
18 red
18 black
= 38 total

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American version / Handout

Casino payouts

Type of Bets And ~~Winning Chances~~

Inside bets

Bet Name	Ex.	Numbers to bet on	Payout	~~Chances~~	True odds
Straight up	A	30	35:1	~~38:1~~	37:1
Split Bet	B	11 or 14	17:1	~~38:2~~	36:2
Street Bet	C	19, 20, 21	11:1	~~38:3~~	35:3
Corner	D	25, 26, 28, 29	8:1	~~38:4~~	34:4
Five Numbers	E	0, 00, 1, 2, 3	6:1	~~38:5~~	
Line Bet	F	4, 5, 6, 7, 8, 9	5:1	~~38:6~~	32:6

Outside Bets

Bet Name	Ex	Numbers to bet on	Payout	~~Chances~~	True odds
Column	G	Set of column numbers	2:1	~~38:12~~	26:12
Dozen	H	25 through 36	2:1	~~38:12~~	26:12
Red or Black	I	Red numbers	1:1	~~38:18~~	20:18
Even or Odd	J	Odd numbers	1:1	~~38:18~~	20:18
Low or High	K	19 through 36	1:1	~~38:18~~	20:18