

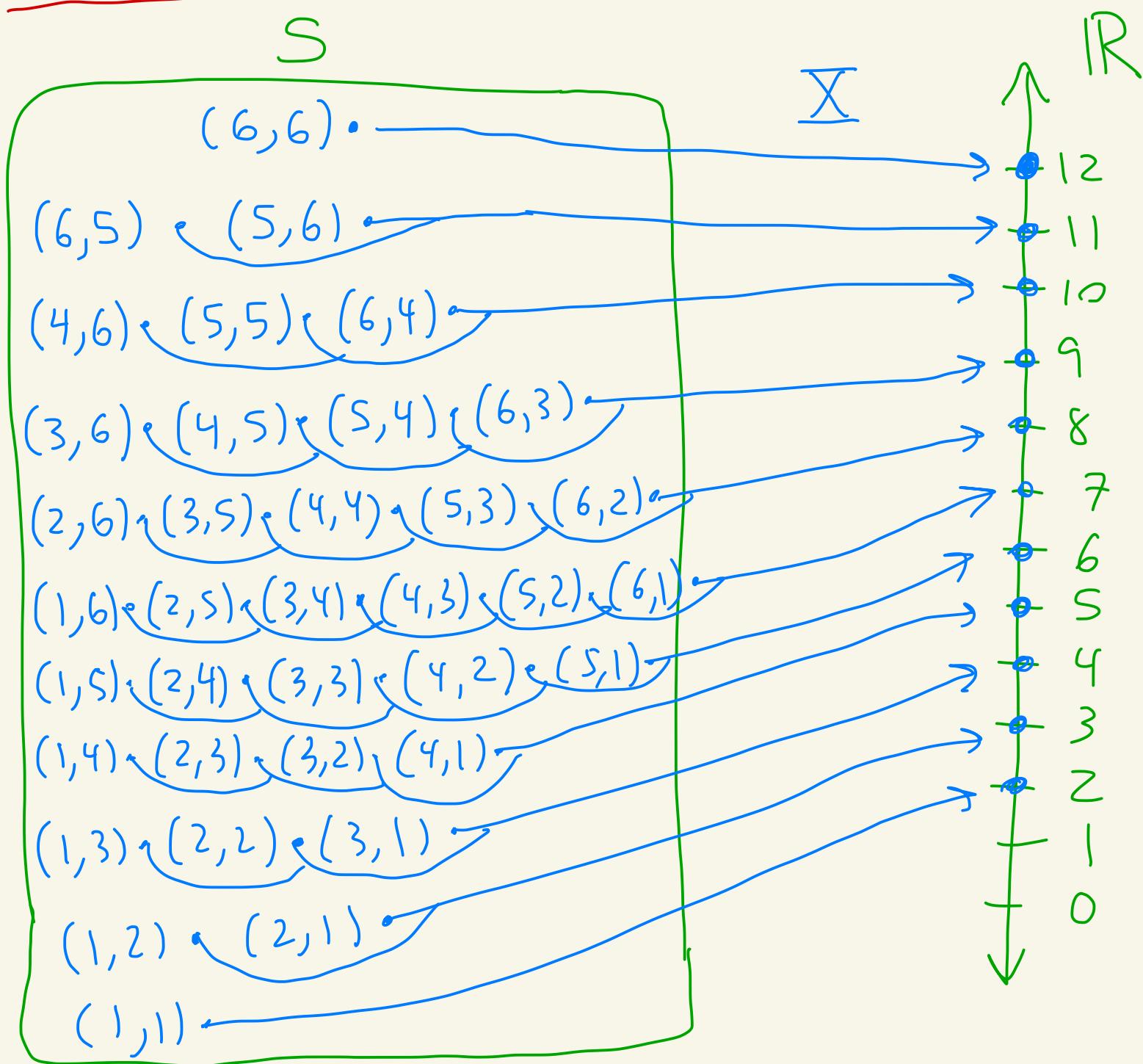
Math 4740

10/16/23



Ex: Let (S, Ω, P) be a probability space corresponding to rolling two 6-sided dice. Let \bar{X} be the sum of the dice.

For example, $\bar{X}(2,5) = 2+5 = 7$



\underline{X} is a discrete random variable.
It's range is 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Def: Let \underline{X} be a random variable on a probability space (S, Ω, P) .

Define:

- $P(\underline{X} = i) = P(\underbrace{\{w \mid w \in S \text{ and } \underline{X}(w) = i\}}_{\substack{\text{set of all outcomes } w \\ \text{where } \underline{X}(w) = i}})$
- $P(\underline{X} \leq i) = P(\underbrace{\{w \mid w \in S \text{ and } \underline{X}(w) \leq i\}}_{\substack{\text{set of all outcomes } w \\ \text{where } \underline{X}(w) \leq i}})$

Similar defs can be made for
 $P(\underline{X} < i), P(\underline{X} \geq i)$, etc.

- The probability function p of \underline{X} is
 - $p(i) = P(\underline{X} = i)$
 - so, $p: \mathbb{R} \rightarrow \mathbb{R}$

• The cumulative distribution function of \bar{X} is

$$F(i) = P(\bar{X} \leq i)$$

So, $F: \mathbb{R} \rightarrow \mathbb{R}$

Ex: Consider the previous example
where (S, Ω, P) represents rolling
two 6-sided dice and
 \bar{X} is the sum of the dice.

Let's draw the probability
function $p: \mathbb{R} \rightarrow \mathbb{R}$ and
cumulative distribution
function $F: \mathbb{R} \rightarrow \mathbb{R}$

Let's calculate.

$$P(2) = P(\bar{X} = 2) = P(\{(1,1)\}) = 1/36$$

$$P(3) = P(\bar{X} = 3)$$

$$= P(\{(1,2), (2,1)\}) = 2/36$$

$$P(4) = P(\bar{X} = 4)$$

$$= P(\{(1,3), (2,2), (3,1)\}) = 3/36$$

$$P(5) = 4/36$$

$$P(6) = 5/36$$

$$P(7) = 6/36$$

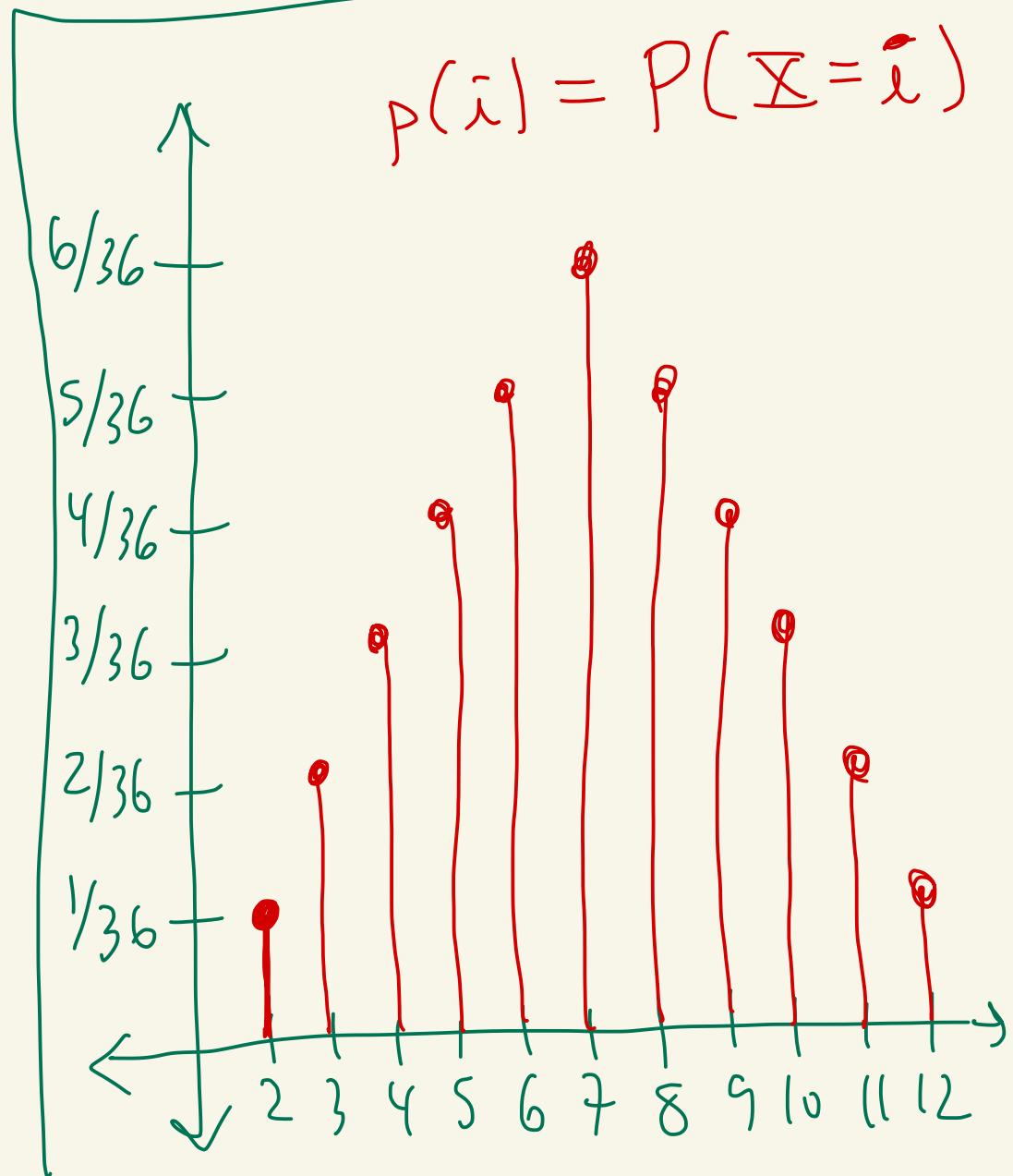
$$P(8) = 5/36$$

$$P(9) = 4/36$$

$$P(10) = 3/36$$

$$P(11) = 2/36$$

$$P(12) = 1/36$$



What about F ?

empty set

$$F(1) = P(\bar{X} \leq 1) = P(\emptyset) = 0$$

$$F(2) = P(\bar{X} \leq 2)$$

$$= P(\{(1,1)\}) = 1/36$$

$$F(3) = P(\bar{X} \leq 3)$$

$$= P(\{(1,1), (1,2), (2,1)\}) = 3/36$$

OR

$$F(3) = P(\bar{X}=2) + P(\bar{X}=3) = 1/36 + 2/36 \\ = 3/36$$

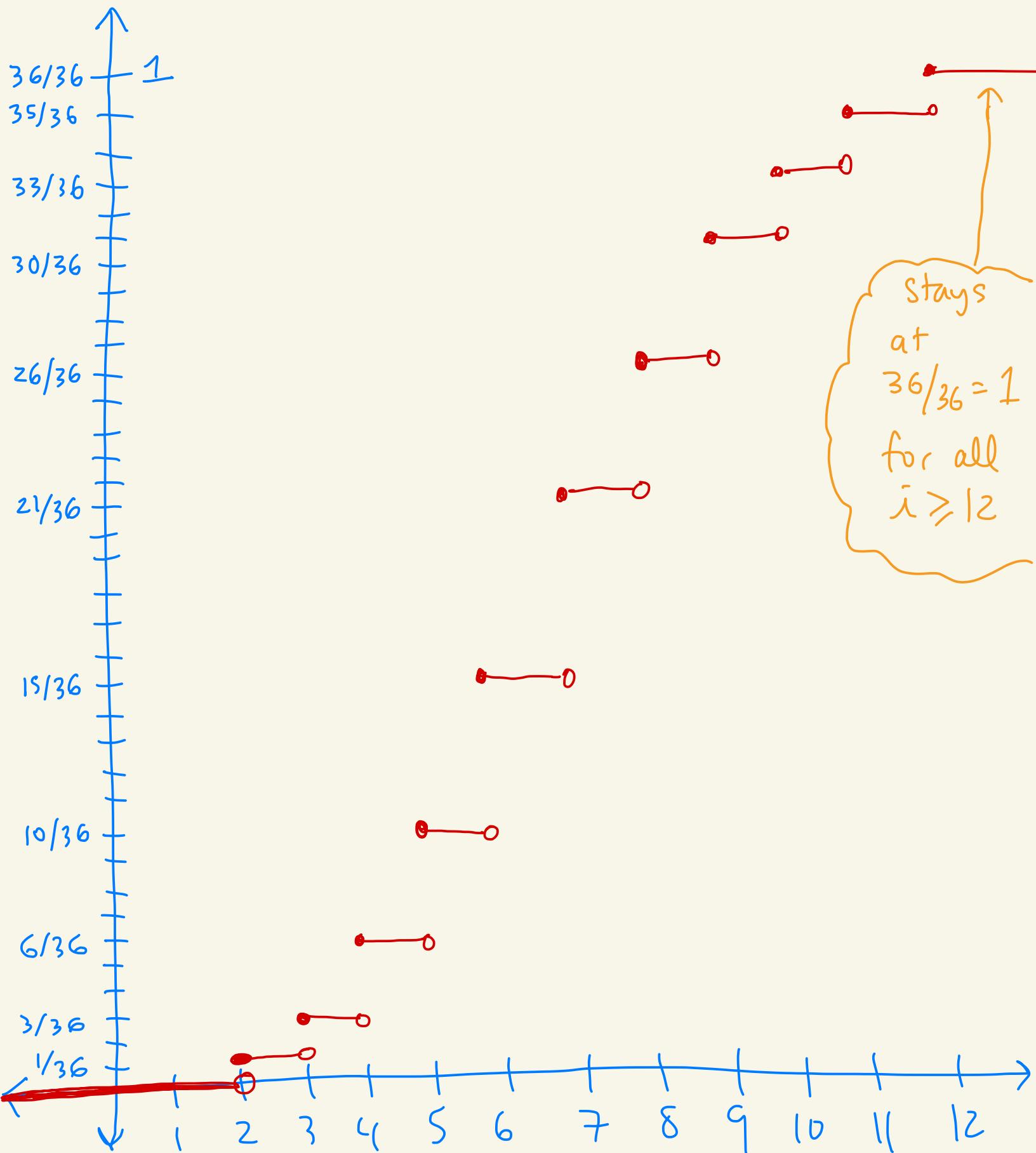
$$F(3.7) = P(\bar{X} \leq 3.7)$$

$$= P(\bar{X} \leq 3) = 3/36$$

$$F(4) = P(\bar{X} \leq 4) = P(\bar{X}=2) + P(\bar{X}=3) + P(\bar{X}=4) \\ = 1/36 + 2/36 + 3/36 = 6/36$$

and so on...

GRAPH OF $F(\bar{x}) = P(\bar{X} \leq \bar{x})$



Def: Let \bar{X} be a discrete random variable on a probability space (S, Ω, P) . The expected value of \bar{X} is

$$E[\bar{X}] = \sum_{w \in S} \bar{X}(w) \cdot P(\{w\})$$

sum over
 outcomes w in S

or if the range of \bar{X} is x_1, x_2, x_3, \dots
 then we get

$$\begin{aligned} E[\bar{X}] &= \sum_i x_i \cdot P(\bar{X} = x_i) \\ &= \sum_i x_i \cdot P(x_i) \end{aligned}$$

$P(x_i)$
 $= P(\bar{X} = x_i)$

Ex: Let's use the same example as before where we roll two 6-sided die and \bar{X} is the sum of the dice.

Long way to calculate $E[\bar{X}]$:

$$E[\bar{X}] = \sum_{\omega \in S} \bar{X}(\omega) \cdot P(\{\omega\})$$

$$= \underbrace{\bar{X}(1,1)}_2 \cdot \underbrace{P(\{(1,1)\})}_{1/36} + \underbrace{\bar{X}(1,2)}_3 \cdot \underbrace{P(\{(1,2)\})}_{1/36}$$

$$+ \underbrace{\bar{X}(2,1)}_3 \cdot \underbrace{P(\{(2,1)\})}_{1/36} + \underbrace{\bar{X}(1,3)}_4 \cdot \underbrace{P(\{(1,3)\})}_{1/36}$$

$$+ \underbrace{\bar{X}(2,2)}_4 \cdot \underbrace{P(\{(2,2)\})}_{1/36} + \underbrace{\bar{X}(3,1)}_4 \cdot \underbrace{P(\{(3,1)\})}_{1/36}$$

$$+ \dots + \underbrace{\bar{X}(6,6)}_{12} \cdot \underbrace{P(\{(6,6)\})}_{1/36}$$

$$= 2\left(\frac{1}{36}\right) + (3+3)\left(\frac{1}{36}\right) + (4+4+4)\left(\frac{1}{36}\right) + \dots + (12)\left(\frac{1}{36}\right)$$

$$= \frac{1}{36} \left[z + z(3) + 3(4) + 4(5) + 5(6) \right. \\ \left. + 6(7) + 5(8) + 4(9) \right. \\ \left. + 3(10) + 2(11) + 1(12) \right]$$

$$= 7$$