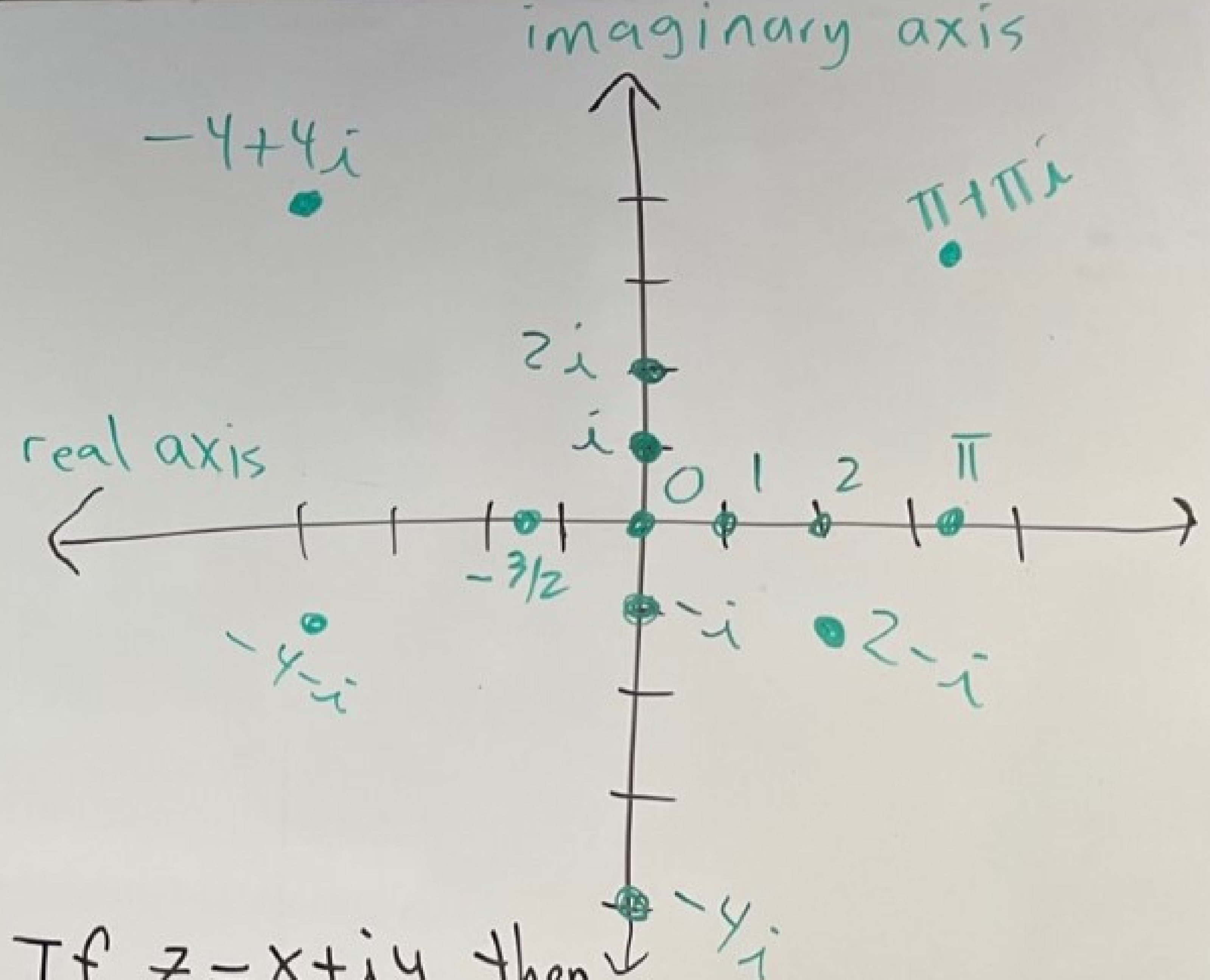
# Complex numbers

**Def:** Define $i$ to be a number that satisfies $i^2=-1$. Define the set of complex numbers to be

$$\mathbb{C}=\{x+iy \mid x,y\in\mathbb{R}\}$$

(real numbers)




If $z=x+iy$, then we write

$\mathrm{Re}(z)=x$ real part of $z$

$\mathrm{Im}(z)=y$ imaginary part of $z$

## Adding

$$(x+iy)+(a+ib) = (x+a)+i(y+b)$$

Ex: $(1-i)+\left(\frac{1}{2}+3i\right) = \left(1+\frac{1}{2}\right)+i(-1+3) = \frac{3}{2}+2i$

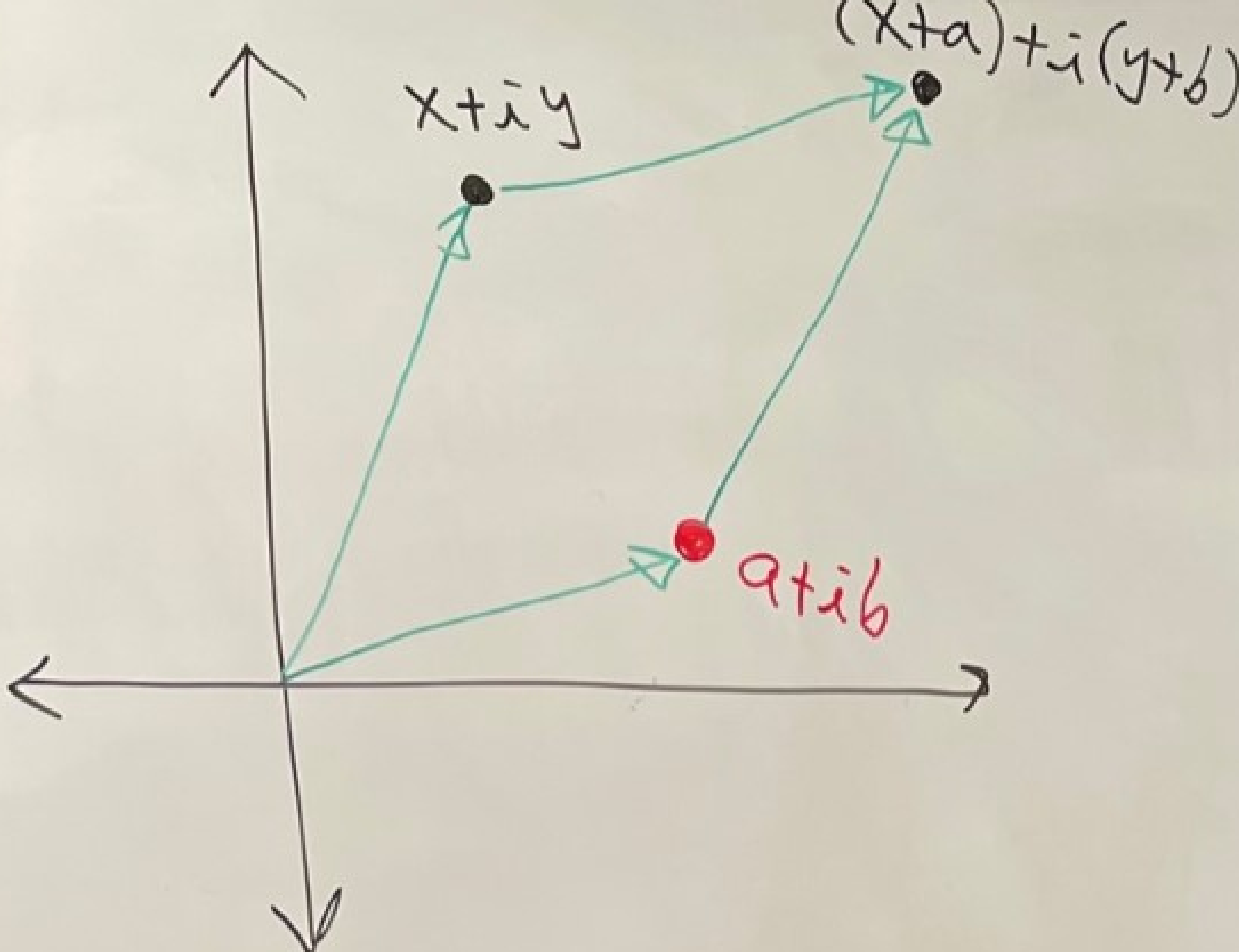
## Multiplying

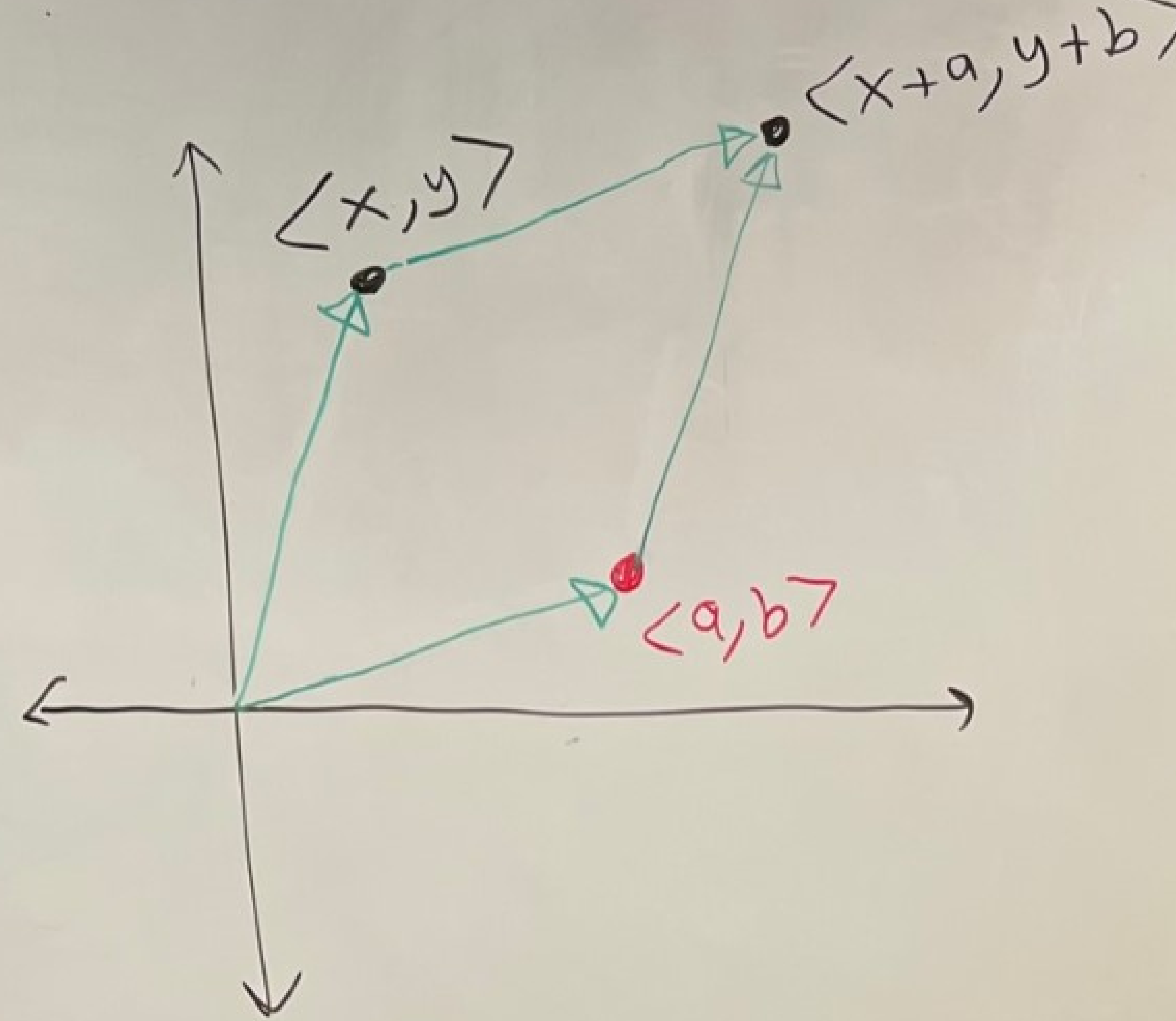
Foil and use $i^2=-1$

$i^2=-1$

Ex: $(1-i)\left(\frac{1}{2}+3i\right) = \frac{1}{2}+3i-\frac{1}{2}i-3i^2 = \frac{1}{2}+\frac{5}{2}i+3 = \frac{7}{2}+\frac{5}{2}i$

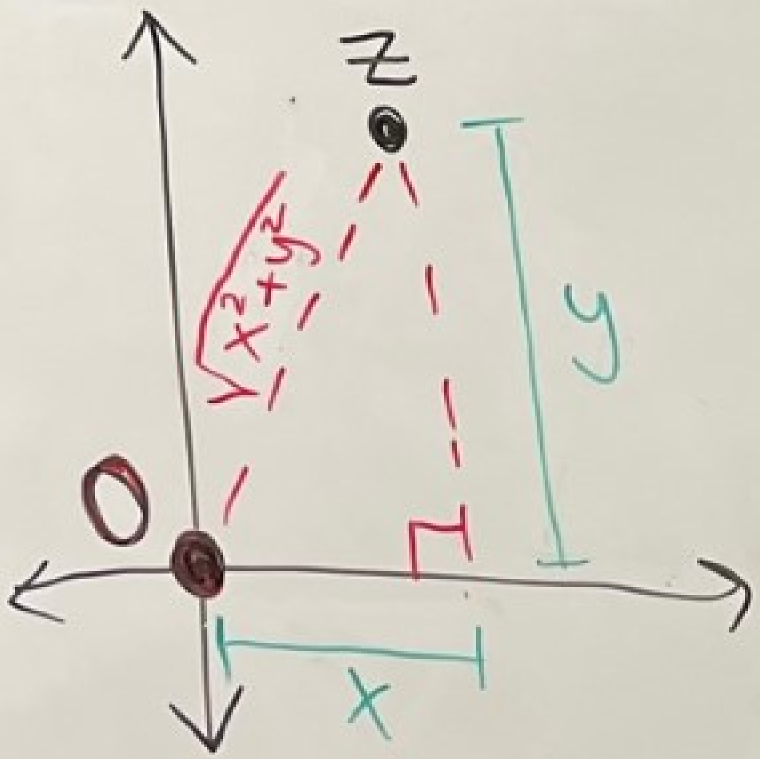
One can think of adding complex numbers like adding vectors.



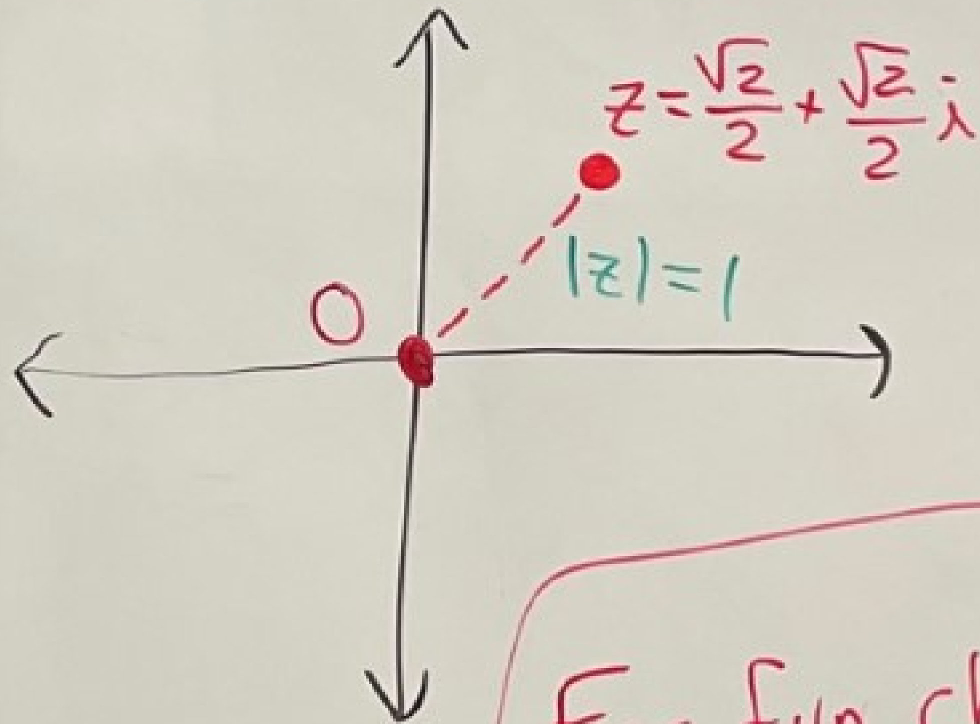



**Def:** Let $z = x + iy$ be a complex number. Then the norm, or absolute value, of $z$ is $|z| = \sqrt{x^2 + y^2}$

This is the distance between $0$ and $z$.




**Ex:** $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$




$$|z| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2}$$

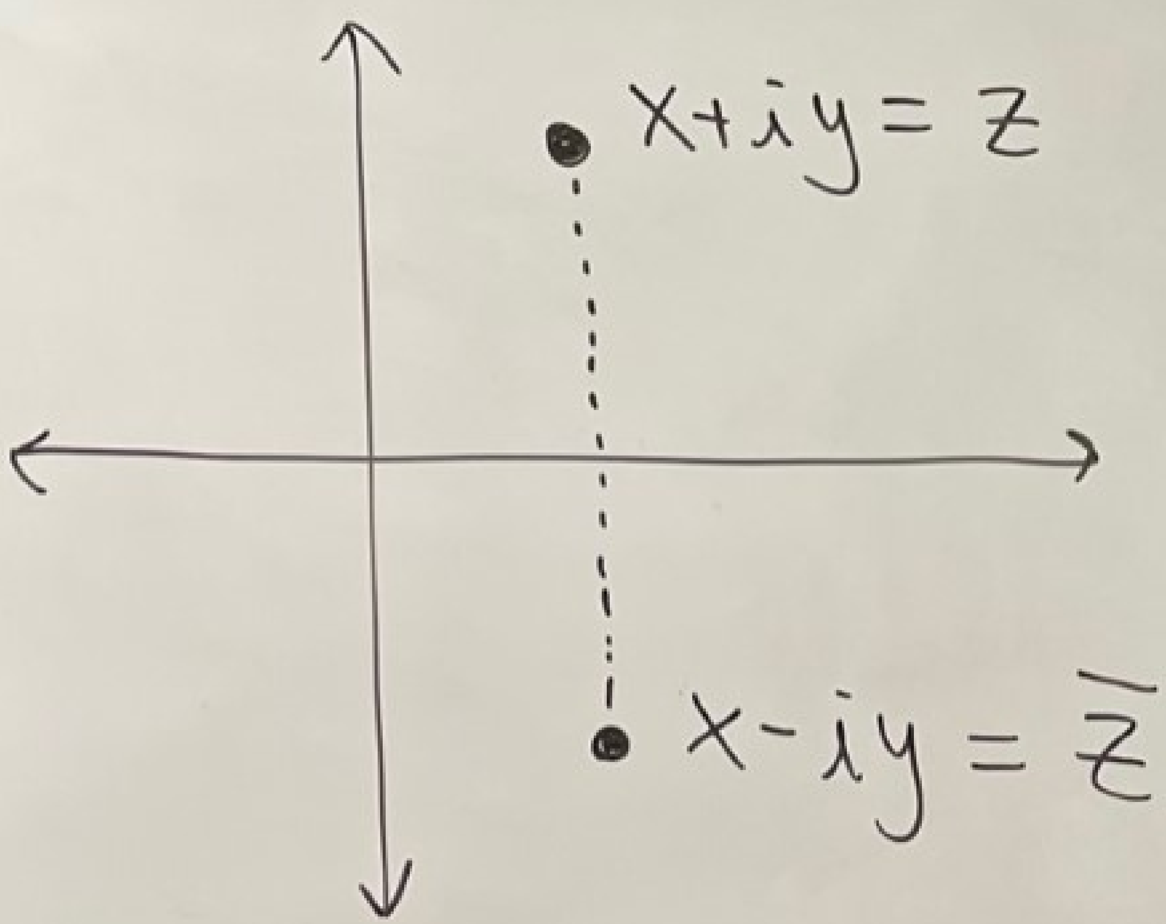
$$= \sqrt{\frac{1}{2} + \frac{1}{2}}$$

$$= 1$$

$\boxed{\text{For fun check: } \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)^2 = i}$

__Def:__ Let $z = x + iy$ be a complex number. The __conjugate__ of $z$ is $\overline{z} = x - iy$.

Another notation is $z^*$




__Ex:__ $\overline{\frac{1}{2} + 5i} = \frac{1}{2} - 5i$

$\overline{10 - 3i} = 10 + 3i$

$\overline{5} = \overline{5 + 0i} = 5 - 0i = 5$