# CALIFORNIA STATE UNIVERSITY <br> LOS ANGELES 

Department of Electrical and Computer Engineering
EE-2449 Digital Logic Lab

EXPERIMENT 9
EXCLUSIVE-OR CIRCUITS AND PARITY

Text: Mano and Ciletti, Digital Design, $5^{\text {th }}$ Edition, Chapters 4 and 5
Required chips: 7486: quad 2-input XOR (3 each).
9.1 All-XOR Circuits: (For ease of use, we replace the exclusive-or symbol $\oplus$ in the following discussion with the keyboard symbol ^. This is common practice in software packages designed for programmable logic devices (PALs, PLDs etc.).

An all-XOR function has the form $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D} \ldots)=\mathrm{A}{ }^{\wedge} \mathrm{B}^{\wedge} \mathrm{C}^{\wedge} \mathrm{D}^{\wedge} \ldots$. At any given time, some of the inputs will be 1 's, the others 0 's; e.g. 01110110 for an 8 -input function. If we group the 1 's and 0 's separately and then pair them, we get $0(00)(11)(11) 1$. This pattern has odd parity (an odd number of 1 's).

Now, the XOR of 2 bits is 0 if they are equal ( 00 or 11 ), and 1 if they are not ( 01 or 10 ). Thus, if we XOR the paired 1 's and 0 's in $0(00)(11)(11) 1$, we get $0(0)(0)(0) 1$. We can rewrite this as $(00)(00) 1$ and again XOR the pairs. The result is $F=0^{\wedge} 0^{\wedge} 1=0^{\wedge} 1=1$. If the extra 1 had instead been a 0 , the result would have been $\mathrm{F}=0^{\wedge} 0=0$.

This will be true in general, regardless of how many inputs there are. If input parity is odd, there is an extra 1 that can't be paired off. The result is always $F=1$. For even parity, it is always $F=0$. Because of this property, XOR's are used in circuits that recognize input parity; i.e. parity generators and checkers.

## 9.2* Parity Generating and Checking Circuits using XORs

Parity is a low cost method for detecting errors in binary data. Stored or transmitted data is susceptible to errors due to radiation or faulty circuits. An error can cause one or more bits of data to change their value. For example, assume that a 7-bit ASCII text character is being transmitted from one cell phone to another. Assume the character being sent is the letter 'A'. The 7-bit ASCII code for 'A' is 1000001. During transmission, assume that radiation causes the least significant bit to change from a 1 to a 0 .

The data being received would be $100000 \underline{0}$ which is the '@' character. Parity can be used to detect that type of error so that the receiver knows that the data is incorrect.

Parity is based on the number of ones in the data to be transmitted; the data's parity is said to be even if the data has an even number of ones; otherwise, parity is odd. The sender adds a parity-bit to the data using a parity generator so that the transmitted combination (parity bit plus data) has the kind of parity the receiver expects. If the receiver expects the combination to have even parity but the data has odd parity, the sender adds a parity-bit of one; if the data has even parity, it adds a zero. Either way, the transmitted combination must have an even number of ones. The opposite is true for odd parity.

Consider the above example where the data is the 7-bit ASCII character 'A'. Suppose sender and receiver are using an even parity system. If a parity bit is placed at the left of the data, then the 8 -bit combination transmitted is $\mathbf{0 1 0 0 0 0 0 1}$. The parity bit is a zero since the data has two ones which is already an even number of ones. If the same error occurs in transmission, the data received will be $\mathbf{0 1 0 0 0 0 0} \mathbf{0}$. At the receiver a parity checker checks the number of ones to make sure the received combination has the correct parity (in this case, even). For this example, the parity checker will detect that there is an odd number of ones (only one 1 ) and flag an error $(\mathrm{C}=1)$. A system like this cannot detect all errors as you'll discover in this experiment nor can it correct the error. But with only one added bit, it can detect a large number of errors. More sophisticated error correcting codes can be used if errors need to be corrected.

## Drawing and Building the circuit

Using Express SCH, draw the following sender-receiver circuit. Design it to use EVEN parity. Replace the Parity Generator and Checker boxes with appropriate XOR circuits. Also, show the Error Simulator as 4 XOR gates (this part is discussed on the next page). Your diagram will consist of nothing but 7486 XOR gates from 3 chips--be sure to include $U$ numbers so you will know which gates come from which chip..

In your drawing, show $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ and Error Simulator inputs, $\mathrm{Sp}, \mathrm{Sx}, \mathrm{Sy}$, and Sz , with input port symbols. Same for $\mathrm{X}^{*}, \mathrm{Y}^{*}, \mathrm{Z}^{*}, \mathrm{P}$, and C, but with output port symbols. (See port symbols for X and $\mathrm{X}^{*}$ below.)


SENDER
ERROR SIMULATOR
RECEIVER

Since PXYZ patterns are to be sent with EVEN parity, the parity generator will examine the input pattern XYZ and output a 1 if the pattern has the wrong parity (ODD), and a 0 otherwise. Thus, if inputs $\mathrm{XYZ}=$ 100 (odd parity), then $\mathrm{P}=1$. The transmitted pattern will be $\mathrm{PXYZ}=1100$, with even parity as required. Again, the parity generator is to be designed with XOR's only.

Like the parity generator, the parity checker also examines its input pattern (in this case $\mathrm{P}^{*}, \mathrm{X}^{*}, \mathrm{Y}^{*}, \mathrm{Z}^{*}$ ) and outputs a 1 if the pattern has the wrong (ODD) parity. It too is to be designed with XORs only.

It is possible for one or more errors to enter the bit pattern as it travels the distance from sender to receiver. (This distance might be only a few centimeters or it might be many miles.) The effect of such errors on the parity checker can be demonstrated by means of the Error Simulator.

This circuit consists of 4 XOR gates used as selectable inverters. Each has a data input and a select input, S . When $\mathrm{S}=1$, the data bit is complemented; otherwise it is unchanged. Thus, $\mathrm{P}^{*}$ $=P^{\prime}$ if $S p=1$, but $P^{*}=P$ if $S p=0$. (See Fig. 4.13, the 4-bit Adder-Subtractor in Mano, for another example of selectable inverters.)

Connect XYZ to three switches (see above diagram)

Truth table showing how XOR gates can be used as selectable inverters: $P^{*}=S \oplus P$

| S | P | P* |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | if $S=0$, |
| 0 | 1 | 1 | $\mathrm{P}^{*}=\mathrm{P}$ |
| 1 | 0 | 1 | if $S=1$ |
| 1 | 1 | 0 | $\mathrm{P}^{*}=\mathrm{P}^{\prime}$, | and connect the $S$ inputs to four switches. Test your circuit with all $S$ inputs initially at 0 . This simulates normal conditions where patterns are transmitted without error; i.e. $\mathrm{P}^{*}=\mathrm{P}, \mathrm{X}^{*}=\mathrm{X}$, etc.. For each of the 8 patterns of XYZ (000-111) observe the LEDs for P and C. Whenever XYZ has odd (wrong) parity (e.g. $X Y Z=010)$, the parity generator should output $\mathrm{P}=1$ so transmitted parity is even $(1010)$. Otherwise, $\mathrm{P}=$ 0 . C should be low all the time, indicating no errors.

Now, introduce an error by setting Sz to logic-1. This inverts $Z^{*}$ and gives the received pattern the wrong parity. C responds by going high (error). Make sure this happens in your circuit. Next, set $S y=S z=1$. Note that C goes low, indicating no error when there actually are two.

Now, set $S x=S y=S z=1$. This should bring C high again. Finally, with all S's high, C goes low once more, even though there are now 4 errors! You can see the limitation of this sort of parity system. It only works reliably if the probability of more than 1 error is very low. Then the chance of 2 or 4 errors is vanishingly small, making it extremely unlikely that C will give a false indication.

Demonstrate all this to your instructor so he/she can sign off on this experiment.

