Assignment 4 Math 570A Fall 2009

1) Let $A, B \in \mathbb{C}^{n \times n}$. Recall $\lambda(A)$ is the set of eigenvalues of A. Show that $\lambda(AB) = \lambda(BA)$.

2) For the matrix

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$$

find a matrix X such that $X^{-1}AX$ is diagonal with the eigenvalues of A on the diagonal (see example 7.1.4)

3) Do 3 iterations of the power method, one iteration of the Rayleigh quotient method, and one iteration of the QR method for finding eigenvalues using the matrix and initial vector

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix} \qquad q_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

4) Let $X \in \mathbb{R}^{n \times n}$ be nonsingular.
(a) Show that $\|A\|_X = \|X^{-1}AX\|_p$ is a matrix norm.
(b) Show that $\|AB\|_X \leq \|A\|_X \|B\|_X$.
(c) Show that for a diagonal matrix $D \in \mathbb{R}^{n \times n}$ we have that $\|D\|_p = \max_{1 \leq i \leq n} |d_{ii}|$
(d) For a nondegenerate $A \in \mathbb{R}^{n \times n}$ find a nonsingular X such that $\|A\|_X = \max_{\lambda \in \lambda(A)} |\lambda|$