Final Math 402A Spring 2006

Name ______________________________

1) Let $\mathbf{F} = [4x^3y^2, x^2 - y^2, yz]$. Compute div$\mathbf{F}$ and curl$\mathbf{F}$.

2) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F} = [xy, z^2, -y^2]$ and $\mathbf{r}(t) = [t, t^2, t^3]$ $0 \le t \le 1$.

3) For the integral

$$\int_{(1,1,2)}^{(2,1,-1)} (12xy + yz)dx + (6x^2 + xz)dy + xydz$$

show that the integrand is an exact form and evaluate the integral.

4) Use Green's theorem to evaluate the integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ over the rectangle with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ where $\mathbf{F} = [y^2, x^2]$.

5) Without using the divergence theorem compute directly the flux integral $\int\int_S \mathbf{F} \cdot \mathbf{n} dA$ where $\mathbf{F} = [6z, -2x, y^2]$ and the surface S is given by $\mathbf{r}(u,v) = [u, v, u^2 + v^2]$ $0 \le u \le 1$, $0 \le v \le 1$

7) Let the contour C be the unit circle tranversed once counterclockwise (parametrized as $z(t) = \cos t + i \sin t$ with $0 \le t \le 2\pi$). Evaluate the integral $\oint_C \text{Re} z \, dz$

8) Let the contour C be the circle $|z| = 2$ transversed once counterclockwise. Evaluate the following integrals

(a) $\oint_C e^z dz$

(b) $\oint_C \frac{z^2 + e^z}{z(z-3)} dz$

(c) $\oint_C \frac{\cos z}{(z-1)^3} dz$

1) Compute the directional derivative $D_a f|_P$, where $a = [1,2,3]$, $f(x,y,z) = xy - yz^2 + xyz$, and we have the point $P(1,1,2)$.

2) Compute the line integral $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$, where $\mathbf{F} = [-y, x, z]$, C: $\mathbf{r}(t) = [\cos t, \sin t, t]$, and we go from the point $(1,0,0)$ to the point $(1,0,4\pi)$.

3) Compute and simplify i^{3+i}

4) Let $\mathbf{F} = [y, 0, z]$ and consider the surface $S\colon \mathbf{r}(u,v) = [u\cos v, u\sin v, u^2]$ with $0 \le u \le 2$ and $0 \le v \le 2\pi$. Compute the surface integral $\int\int_S \mathbf{F}\cdot\mathbf{n}\,dA$.

5) Show the following integral is path independent and compute its value.

$$\int_{(1,1,0)}^{(2,4,\pi)} ((\cos z + yz)dx + xzdy + (xy - x\sin z + 2z)dz)$$

6) Let $\mathbf{F} = [xy, y^2, zy]$. Use the divergence theorem to compute the surface integral $\int\int_S \mathbf{F}\cdot\mathbf{n}\,dA$ where S is the surface of the cyllinder $x^2 + y^2 \le 4$ and $-3 \le z \le 3$. (Hint: Use polar coordinates for x and y when you do the integral, ie use cyllindrical coordinates).

7) Consider $C\colon |z| = 2$ transversed counterclockwise once. Evaluate the following integrals.

(a) $\oint_C \operatorname{Re} z\,dz$

(b) $\oint_C \sin z\,dz$

(c) $\oint_C \dfrac{z^3 + e^z}{z(z-3)}\,dz$

1) Compute the directional derivative of $f(x,y,z) = x^2 - 3xy + 4z^3$ in the direction of $[1,2,2]$ at the point $P(3,4,1)$.

2) Let $\mathbf{F} = [2x^2y^2, y, yz]$. Compute div$F$ and curl$F$.

3) Calculate $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = [z, xy, yz]$ and $\mathbf{r}(t) = [t, t^2, t]$, $0 \le t \le 1$.

4) For the integral

$$\int_{(0,1,2)}^{(2,1,-8)} 2xydx + (x^2 + z)dy + ydz$$

show that the integrand is an exact form then evaluate the integral (you need to find the potential).

5) Without using the divergence theorem compute directly the flux integral $\int\int_S \mathbf{F}\cdot\mathbf{n}dA$ where $\mathbf{F} = [x, y, z]$ and S is the surface given by $\mathbf{r}(u,v) = [\cos u, \sin u, v]$, $0 \le u \le 2\pi$ and $0 \le v \le 1$

6) Use Green's theorem to evaluate the path integral $\oint_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = [3x+4y, 2x+3y^2]$ and C is the circle $x^2 + y^2 = 1$ transversed counterclockwise.

7) Use the divergence theorem to evaluate $\int\int_S \mathbf{F}\cdot\mathbf{n}dA$ where $\mathbf{F} = [xy, y^2z, z]$ and S is the surface of the cube $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$.

8) Show $f(z) = z^2 + z$ is analytic using the Cauchy-Riemann equations (hint: put it in the form $u(x,y) + iv(x,y)$)

9) Show that $u(x,y) = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.