Math 206 Midterm II Fall 2009
SHOW YOUR WORK

Name ______________________________

(1) (4 points each) Evaluate the following limits using L'Hospital's rule

(a) $\lim_{x\to 1} \frac{\ln x}{x-1}$ $\overset{L}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{1} = 1$

(b) $\lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$ $\overset{L}{=} \lim_{x\to 0} \frac{e^x - e^{-x}}{2\sin 2x} \overset{L}{=} \lim_{x\to 0} \frac{e^x + e^{-x}}{4\cos 2x} = \frac{1}{2}$

(c) $\lim_{x\to 0+} x^3 \ln x$ $= \lim_{x\to 0^+} \frac{\ln x}{x^{-3}} \overset{L}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-3x^{-4}}$

$= \lim_{x\to 0^+} \frac{x^4}{-3x} = \lim_{x\to 0^+} \frac{x^3}{-3} = 0$

(2) (4 points each) Differentiate the following functions

(a) $f(x) = 3^{\cos x}$

$$f'(x) = 3^{\cos x} \ln 3 (-\sin x)$$

(b) $f(x) = \ln x 10^x$

$$\frac{1}{x} 10^x + \ln x\, 10^x \ln 10$$

(c) $f(x) = \dfrac{6^{(x^2+x)} + x}{x+1}$

$$f'(x) = \frac{[6^{(x^2+x)} \ln 6 (2x+1)](x+1) - [6^{(x^2+x)} + x]}{(x+1)^2}$$

(d) $f(x) = \tan^{-1}(x^2+1)$

$$f'(x) = \frac{1}{1+(x^2+1)^2} 2x = \frac{2x}{x^4+2x^2+2}$$

(e) $f(x) = \sin^{-1}(\sqrt{1-x^2})$

$$f'(x) = \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}} \frac{1}{2} \frac{1}{\sqrt{1-x^2}} (-2x)$$

$$= \frac{-x}{\sqrt{1-1+x^2}} \frac{1}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{x^2}} \frac{1}{\sqrt{1-x^2}} = -\frac{1}{\sqrt{1-x^2}}$$

if $x \geq 0$
so that $\sqrt{x^2} = x$

(3) **(5 points each) Find all the critical points of the following functions**

(a) $f(x) = x^3 + 3x^2 - 9x + 1$

$$f'(x) = 3x^2 + 6x - 9 = 3\cdot(x^2 + 2x - 3)$$

$$= 3(x+3)(x-1)$$

$$x = -3, 1$$

(b) $f(x) = \dfrac{x}{x^2+2}$

$$f'(x) = \frac{(1)(x^2+2) - x(2x)}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}$$

$$x = \pm\sqrt{2}$$

(c) $f(x) = x^{4/3} - 6x^{1/3}$

$$f'(x) = \frac{4}{3}x^{1/3} - 6\left(\frac{1}{3}\right)x^{-2/3}$$

$$= \frac{4x^{1/3}}{3} - \frac{6}{3x^{2/3}} = \frac{4x^{1/3}}{3}\frac{x^{2/3}}{x^{2/3}} - \frac{6}{3x^{2/3}}$$

$$= \frac{4x-6}{3x^{2/3}} \qquad x = 0 \qquad 4x - 6 = 0$$

$$x = \frac{6}{4}$$

critical points are $x = 0, \frac{3}{2}$

(4) (12 points) Find the absolute maximum and absolute minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$ on the interval $[-2, 3]$

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$

$x = 2, -1$

$f(-1) = -2 - 3 + 12 = 7$ abs. max.

$f(2) = 16 - 12 - 24 = -20$ abs. min

$f(-2) = -16 - 12 + 24 = -4$

$f(3) = 54 - 27 - 36 = 27 - 36 = -9$

(5) (12 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ for

$y = \frac{x^4 \sqrt[5]{10x - 20}}{(1 + x^3)^4}$

$\ln y = \ln\left(\frac{x^4 \sqrt[5]{10x-20}}{(1+x^3)^4}\right) = \ln x^4 + \ln \sqrt[5]{10x-20} + \ln (1+x^3)^4$

$= 4 \ln x + \frac{1}{5} \ln (10x - 20) + 4 \ln (1+x^3)$

$\frac{1}{y} y' = \frac{4}{x} + \frac{1}{5} \frac{1}{10x-20}(10) + \left(\frac{4}{1+x^3}\right)(3x^2)$

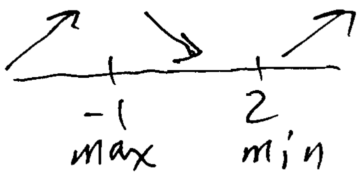
$y' = y\left[\frac{4}{x} + \frac{2}{10x-20} + \frac{12x^2}{1+x^3}\right]$

$= \frac{x^4 \sqrt[5]{10x-20}}{(1+x^3)^4}\left[\frac{4}{x} + \frac{1}{5x-10} + \frac{12x^2}{1+x^3}\right]$

(6) (15 points) For the function $f(x) = 2x^3 - 3x^2 - 12x + 9$ find where
(a) its increasing and decreasing
(b) local maximums and local minimums are
(c) it is concave upward and concave downward
(d) inflection points are
(e) lastly graph it

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$$

$x = 2, -1$ -1 2



	$6(x-2)$	$x+1$	f'
$(-\infty, -1)$	$-$	$-$	$+$ increasing
$(-1, 2)$	$-$	$+$	$-$ decreasing
$(2, \infty)$	$+$	$+$	$+$ increasing

$$f''(x) = 12x - 6 = 0$$

$x = \frac{1}{2}$ $(-\infty, \frac{1}{2})$ concave down

$\cancel{f(\frac{1}{2})}$ $(\frac{1}{2}, \infty)$ concave up

$$f(\tfrac{1}{2}) = \frac{2}{8} - \frac{3}{4} - \frac{12}{2} + \frac{18}{2}$$
$$= \frac{1}{4} - \frac{3}{4} - \frac{24}{4} + \frac{36}{4}$$
$$= \frac{5}{2}$$ inflection point $(\frac{1}{2}, \frac{5}{2})$

$f(-1) = 16$

$f(2) = -11$

$f(0) = 9$