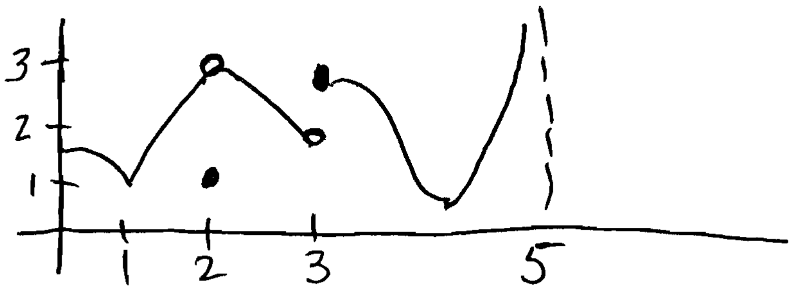
Math 206 Midterm Fall 2009
SHOW YOUR WORK

Name ____

(1) Use the figure below to answer true or false to the following questions



(a) $f(x)$ is continuous at $x = 1$ yes

(b) $f(x)$ is differentiable at $x = 1$ no

(c) $\lim_{x\to 2} f(x) = 3$ yes

(d) $\lim_{x\to 3^+} f(x) = 3$ yes

(e) $f(2) = 3$ no

(f) $\lim_{x\to 3} f(x) = 2$ no (undefined/doesn't exist)

(g) $\lim_{x\to 5} f(x) = \infty$ yes

(2) Compute the following limits

(a) $\lim_{x\to 2} \frac{x^2 - 6x + 8}{x - 2} = \lim_{x\to 2} \frac{(x-2)(x-4)}{x-2} = \lim_{x\to 2}(x-4) = -2$

(b) $\lim_{x\to\infty} \frac{10x^5 - 3x^2 + x}{6x^5 - 3x^3 + x - 1} = \lim_{x\to\infty} \frac{\frac{10x^5}{x^5} - \frac{3x^2}{x^5} + \frac{x}{x^5}}{\frac{6x^5}{x^5} - \frac{3x^3}{x^5} + \frac{x}{x^5} - 1} = \lim_{x\to\infty} \frac{10 - \frac{3}{x^3} + \frac{1}{x^4}}{6 - \frac{3}{x^2} + \frac{1}{x^4} - 1}$

$= \lim_{x\to\infty} \frac{10}{6} = \frac{5}{3}$

(c) $\lim_{x\to 4} \sqrt{x^2 + x + 5} = \sqrt{4^2 + 4 + 5} = \sqrt{25} = 5$

(f) $\lim_{x\to 1^+} \frac{2}{x-1} = \infty$

(3) Compute the derivatives of the following functions. Simplify where possible.

(a) $f(x) = 3x^5 - x^4 - 9x + 1$

$f'(x) = 15x^4 - 4x^3 - 9$

(b) $f(x) = log_2(\cos x - x^3)$

$f'(x) = \frac{1}{(\cos x - x^3)\ln 2}(-\sin x - 3x^2)$

(c) $g(x) = \frac{x^2+1}{x^4+1}$

$g'(x) = \frac{2x(x^4+1) - 4x^3(x^2+1)}{(x^4+1)^2}$

$= \frac{2x^5 + 2x - 4x^5 - 4x^3}{(x^4+1)} = \frac{-2x^5 - 4x^3 + 2x}{(x^4+1)}$

(d) $h(x) = \cos(\sec x + x^4)$

$h'(x) = -\sin(\sec x + x^4)(\sec x \tan x + 4x^3)$

(e) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$

(f) $g(x) = (1+3x^2)^{10}(x^3+2x)^9$

$g'(x) = 10(1+3x^2)^9(x^3+2x)^9 + (1+3x^2)^{10}(9)(x^3+2x)^8(3x^2+2)$

$= (1+3x^2)^9(x^3+2x)^8[10(x^3+2x) + 9(1+3x^2)(3x^2+2)]$

$= (1+3x^2)^9(x^3+2x)^8[81x^4 + 10x^3 + 81x^2 + 20x + 18]$

(4) Using implicit differentiation find y' for $x^4y^3 - 5x^2y^2 + x = 10$

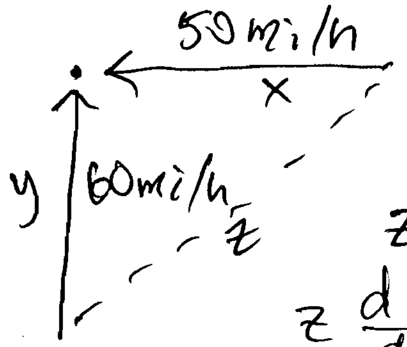
$$4x^3y^3 + x^4 3y^2 y' - 5(2)xy^2 - 5(2)x^2 y y' + 1 = 0$$

$$3x^4y^2y' - 10x^2yy' = 10xy^2 - 4x^3y^3 - 1$$

$$(3x^4y^2 - 10x^2y)y' = 10xy^2 - 4x^3y^3 - 1$$

$$y' = \frac{10xy^2 - 4x^3y^3 - 1}{3x^4y^2 - 10x^2y}$$

(5) Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the same intersection of two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection.



$\frac{dx}{dt} = -50$ $\frac{dy}{dt} = -60$ what is $\frac{dz}{dt}$

when $x = .3$ $y = .4$

$z^2 = x^2 + y^2$. so $2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$ so $\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$

$z^2 = x^2 + y^2 = (.3)^2 + (.4)^2 = .09 + .16$

$= .25$ so $z = .5$ hence

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) = \frac{1}{.5}\left(.3(-50) + .4(-60)\right)$$

$$= 2(-15 - 24) = 2(-39) = -78$$

so $\frac{dz}{dt} = -78$

(6) Find the equation of the tangent line to $y = 3x^3 - 10x^2 + 2$ at $(1, -5)$.

$y' = 9x^2 - 20x$ when $x = 1$ $y' = 9(1)^2 - 20(1) = -11$

So the line is $y + 5 = -11(x - 1)$

which gives $y = -11x + 6$

(7) Compute the first to the sixth derivatives of $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$

$f'(x) = 5x^4 + 4x^3 + 3x^2 + 2x + 1$

$f''(x) = 20x^3 + 12x^2 + 6x + 2$

$f'''(x) = 60x^2 + 24x + 6$

$f^{(4)}(x) = 120x + 24$

$f^{(5)}(x) = 120$

$f^{(6)}(x) = 0$

(8) Explain how you know that the polynomial $f(x) = x^4 + x^2 + x - 5$ has a zero (a root) in the interval $(1, 2)$. State any theorems you use.

We use the Intermediate Value thereom

$f(1) = 1 + 1 + 1 - 5 = -2$

$f(2) = 16 + 4 + 2 - 5 = 17$

$f(1) = -2 < 0 < 17 = f(2)$ so somewhere between 1 and 2 $f(c) = 0$ is c is in $(1, 2)$ by the intermediate value ~~theor~~ thereom