

Practice Midterm Questions

1) Differentiate the following

(a) $f(x) = 3^x x^2$

(b) $f(x) = e^{\cos x}$

(c) $f(x) = 10^{x^3+x^2+x}$

(d) $f(x) = 5^{x \cos x}$

(e) $\frac{5^x - x}{x^2 + 1}$

(f) $f(x) = x^3 \tan^{-1} x$

(g) $f(x) = \frac{\sin^{-1} x}{x^2 - x}$

(h) $f(x) = 3^{\sin^{-1} x}$

(i) $f(x) = \tan^{-1}(\sqrt{x^2 - 1})$

2) Use logarithmic differentiation to find $\frac{dy}{dx}$ for the following

(a) $y = \frac{x^2 \sqrt[3]{7x-14}}{(1+x^2)^4}$

(b) $y = \frac{(x^2-8)^{1/3} \sqrt{x^3+1}}{x^6-7x+5}$

(c) $y = x^{\cos x}$

3) Use L'Hospital's rule to evaluate the following limits

(a) $\lim_{x \rightarrow 1} \frac{\ln x}{x-1}$

(b) $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$

(c) $\lim_{x \rightarrow 0} \frac{x - \tan x}{\sin x - x}$

(d) $\lim_{x \rightarrow \infty} \frac{x \ln x}{x + \ln x}$

(e) $\lim_{x \rightarrow 0^+} \frac{1 - \ln x}{e^{1/x}}$

(f) $\lim_{x \rightarrow 0^+} x^2 \ln x$

4) Find the absolute maximum and minimum for the following functions on each interval

(a) $f(x) = 2x^3 - 15x^2 + 36x$ on $[1, 5]$

(b) $f(x) = 2x^3 - 3x^2 - 12x$ on $[-2, 3]$

(c) $f(x) = x^4 - 2x^2 + 10$ on $[-2, 3]$

(d) $f(x) = x\sqrt{1-x^2}$ on $[-1, 1]$

(e) $f(x) = 2 - \sqrt[3]{x}$ on $[-1, 8]$

5) For the following functions find where

(i) its increasing and decreasing

(ii) local maximums and minimums are

(iii) it is concave upward and concave downward

(iv) inflection points are

(v) lastly graph it

- (a) $f(x) = x^3 + 3x^2 - 9x$
- (b) $f(x) = 2x^3 - 3x^2 - 12x + 3$
- (c) $f(x) = 8x^5 - 5x^4 - 20x^3$ (you can use the quadratic equation to find the inflection points)

6) Find all the critical numbers for the following functions

- (a) $f(x) = x^3 + x^2 + x$
- (b) $f(x) = \frac{y-1}{y^2-y+1}$
- (c) $f(x) = x^{4/5}(x-2)^2$
- (d) $f(x) = x^3 + 3x^2 - 24x$