**Math 206 Final Winter 2008**

Name ______________________________________________

**Show your work and write neatly**

2) (4 points each) Evaluate the following limits

(a) $\lim_{x\to 2}(x^2+x+1)$

(b) $\lim_{x\to 3}\dfrac{x^2-2x-3}{x-3}$

(c) $\lim_{x\to 2^+}\dfrac{x+3}{x-2}$

(d) $\lim_{x\to\infty}\dfrac{x^3+x+5}{10x^3-6x^2+10}$

(e) $\lim_{x\to 0}\cos x$

3) (4 points each) Differentiate the following functions

(a) $f(x)=10x^4-2x^3-5$

(b) $f(x)=\cos x-\tan x$

(c) $f(x)=x\ln x$

(d) $f(x)=\dfrac{x^2+x}{4x-3}$

(e) $f(x)=\sin^{-1}x+\tan^{-1}x$

(f) $f(x)=(x^2+x)^{10}$

4) (8 points) Use implicit differentiation to find $\dfrac{dy}{dx}$ for $y^3+x^2y+\cos y-\sin x=10$

5) (5 points) Find the equation of the tangent line to $f(x)=x^2+x$ at $(2,6)$.

6) (9 points) At noon, ship A is 60 km west of ship B. Ship A is sailing east at 10 km/h and ship B is sailing north at 15 km/h. How fast is the distance between the ships changing two hours later at 2:00pm.

(b) If $y=x^4+3x^2$ find the differential $dy$ of $y$.

8) (4 points) Use L'Hospital's rule to evaluate the following limits

(a) $\lim_{x\to 0}\dfrac{e^x-1}{x}$

(b) $\lim_{x\to\infty}\dfrac{\ln x}{x^2}$

9) (8 points) Find the absolute maximum and absolute minimum of the function $f(x)=2x^3-3x^2-12x$ on $[-2,3]$

10) (10 points) For the functions $y=f(x)=x^3-3x^2+2$ determine where

(a) it is increasing and decreasing

(b) find the location of any local maximum and local minimum

(c) determine where it is concave up and concave down

(d) determine where any inflection points are.

(e) graph it indicating local maximum(s), local minimum(s), inflection(s), and the $y$-intercept.

11) (4 points each) Find the following antiderivatives

(a) $\int x^3 + x^4 + 10dx$

(b) $\int \cos x dx$

(c) $\int \sqrt{x} + \sqrt[3]{x} dx$

(d) $\int e^x dx$

(2) Compute the following limits (you may need to use L'Hospital's rule)

(a) $\lim_{x\to 1} \frac{x^2 - 6x + 8}{x - 2}$

(b) $\lim_{x\to\infty} \frac{10x^5 - 3x^2 + x}{6x^5 - 3x^3 + x - 1}$

(c) $\lim_{x\to 4} \sqrt{x^2 + x + 5}$

(d) $\lim_{x\to 0} \frac{e^x - 1}{x}$

(e) $\lim_{x\to\infty} \frac{xe^x}{e^x - 10}$

(f) $\lim_{x\to 1^+} \frac{2}{x - 1}$

(4) Compute the derivatives of the following functions. Don't use the definition. Simplify where possible.

(a) $f(x) = 3x^5 - x^4 - 9x + 1$

(b) $g(x) = e^{3x} + \ln x$

(c) $h(x) = e^x \tan x$

(d) $f(x) = 5^{2x^3 - x + 1}$

(e) $g(x) = \frac{x^2 + 1}{x^4 + 1}$

(f) $h(x) = \ln(\tan x + x^4)$

(g) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

(h) $g(x) = (1 + 3x^2)^{10}(x^3 + 2x)^9$

(i) $h(x) = \tan^{-1}(x)$

(5) Using implicit differentiation find $y'$ for $x^4y^3 - 5x^2y^2 + x = 10$

(6) A girl flies a kite at a height of 300 feet, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 feet away from her?

(7) Find the absolute maximum and minimum of the function $f(x) = x^4 - 2x^2 + 10$ on the interval $[-2, 3]$.

(8) Compute the differential of each of the following

(a) $y = x^3 - x^2 + x$

(b) $y = x^5 \tan x$

(9) For the function $f(x) = x^3 + 3x^2 - 24x$ indicate

(a) where it is increasing and decreasing
(b) where the local maximum(s) and minimum(s) are
(c) where it is concave upward and concave downward
(d) where inflection point(s) occur
(e) graph it
(10) Find the equation of the tangent line to $y = (x^3 + 2)e^{2x-2}$ at $x = 1$.
(11) Compute the first to the sixth derivatives of $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$
2) Compute the following limits
(a) $\lim_{x \to 2} (x^3 + 2x^2 - 10) =$
(b) $\lim_{x \to 1} \dfrac{x^2 + 1}{x^2 + 3x + 1} =$
(c) $\lim_{x \to 3} \dfrac{x^2 + 3x - 18}{x^2 - 4x + 3} =$
(d) $\lim_{x \to \infty} \dfrac{10x^4 - x^3 + 4x + 7}{6x^4 + 8x^2 - 23} =$
(e) $\lim_{x \to 2^-} \dfrac{1}{x - 2} =$
(f) $\lim_{x \to \pi/2} \dfrac{1 - \sin x}{\cos x} =$
(g) $\lim_{x \to 0} x \ln x =$
(h) $\lim_{x \to \infty} x^{1/x}$
3) Compute the following derivatives (don't use the definition).
(a) $f(x) = 30$
(b) $f(x) = 12x^4 - 10x^3 + x - 33$
(c) $f(x) = \sqrt{x} - \dfrac{10}{\sqrt{x}}$
(d) $f(x) = 10e^x + x^2$
(e) $f(x) = x^2 \sin x$
(f) $f(x) = \dfrac{x^2 + x + 1}{2x - 10}$
(g) $f(x) = e^{\tan x + x^2}$
(h) $f(x) = 3^{x^2 + x}$
(i) $f(x) = \cos x \sin^{-1} x$
5) Use implicit differentiation to find $\dfrac{dy}{dx}$ for $y^3 + xy^2 + 10x^2y - 7 = 0$
6) Two people leave an intersection at the same time. Person A walks east at 3ft/s. Person B walks south at 4ft/s. How fast is the distance between them changing after 10 seconds?
7) Compute the differential $dy$ for the following functions
(a) $y = x^3 + 10x^2 + 6x$
(b) $y = \tan x \sec x$
8) Find the absolute maximum and absolute minimum of $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[-3, 0]$.
9) For the curve $y = f(x) = x^3 - 3x^2 + 2$ indicate
i) where it is increasing and decreasing

ii) where all local maximums and minimums are
iii) where it is concave up and concave down
iv) where inflection points are
v) make a sketch of the curve indicating these points
10) Find the dimensions of a rectangle with a perimeter of 100ft whose area is as large as possible.
11) Find the equation of the tangent line to
(a) $y = f(x) = 2x^3 + x^2 - 6x - 2$ at $(2, 7)$
(b) $y = f(x) = \dfrac{x}{x+1}$ at $(1, \frac{1}{2})$
12) Compute the first to sixth derivatives of $f(x) = x^5 + 2x^3 - 5x^2 + 10x - 20$
13) Let $\lim_{x \to 2} f(x) = -3$ and $\lim_{x \to 2} g(x) = 5$. Compute the following limits
(a) $\lim_{x \to 2} [f(x) + g(x)]$
(b) $\lim_{x \to 2} \dfrac{g(x)}{f(x) + 4}$
(c) $\lim_{x \to 2} \sqrt[3]{g(x) + 3}$
14) Explain what it means to say that

$$\lim_{x \to 1^-} f(x) = 3 \quad \text{and} \quad \lim_{x \to 1^+} f(x) = 7$$

In this situation is it possible that $\lim_{x \to 1} f(x)$ exists? Explain.