My responses are below, in brackets. - Quimby

Show that if $n \geq 3$ for nonabelian S_n, then the only element of $\sigma \in S_n$ satisfying $\sigma\gamma=\gamma\sigma$ $\forall \gamma \in S_n$ is $\sigma=\iota$.

We know that $n \geq 3$ means 3 or more permutations since we're in S_n. [Don't write this sentence---it's not part of the proof.]

We need to show that $\sigma\gamma=\gamma\sigma$. [No! We are given that $\sigma\gamma=\gamma\sigma$. We need to show that $\sigma=\iota$. This is going to mess up the rest of your proof. I can't remember if you were in class today (Thursday), but if you weren't get the notes from someone---I gave a big hint in class.]

Let $a=\sigma\iota$ s.t. $a\neq\sigma$.

(I've realized that I should have never stated that $a\neq\sigma$, by deft of identity, $a=\sigma$ but $a\neq\iota$. Have to specify this because $\iota\iota=\iota$ is true. Trying to prove that $\sigma=\iota$ not assume it. Does that make sense?)

Also, let $\gamma=\iota$ s.t. $\gamma\iota=\iota$. And, $b=\gamma a$ s.t. $b\neq\gamma$.

Show that $(\sigma\gamma)\iota= \gamma(\sigma\iota)$.

Working on the LHS: $(\sigma\gamma)\iota = \sigma(\gamma\iota)$ by Associativity.

$= \sigma\iota$

$= a$

Working on the RHS: $\gamma(\sigma\iota) = \gamma a$

$= b$

$\therefore a\neq b \Rightarrow \sigma\gamma\neq\gamma\sigma \quad \Rightarrow\Leftarrow$

$\therefore\ \sigma=\iota\ \forall\ \sigma \in S_n$