1. Find the value of $\sin 2a$ when

$$(1)\ \cos a=\frac{3}{5},\quad (2)\ \sin a=\frac{12}{13},\ \text{and}\ (3)\ \tan a=\frac{16}{63}.$$

2. Find the value of $\cos 2a$ when

$$(1)\ \cos a=\frac{15}{17},\quad (2)\ \sin a=\frac{4}{5},\ \text{and}\ (3)\ \tan a=\frac{5}{12}.$$

Verify by a graph and accurate measurement.

3. If $\tan\theta=\frac{b}{a}$, find the value of $a\cos 2\theta+b\sin 2\theta$.

Prove that

25. $\sin 3A+\sin 2A-\sin A=4\sin A\cos\frac{A}{2}\cos\frac{3A}{2}$.

26. $\tan 2A=(\sec 2A+1)\sqrt{\sec^2 A-1}$.

27. $\cos^3 2\theta+3\cos 2\theta=4(\cos^6\theta-\sin^6\theta)$.

28. $1+\cos^2 2\theta=2(\cos^4\theta+\sin^4\theta)$.

29. $\sec^2 A(1+\sec 2A)=2\sec 2A$.

30. $\operatorname{cosec} A-2\cot 2A\cos A=2\sin A$.

31. $\cot A=\frac{1}{2}\left(\cot\frac{A}{2}-\tan\frac{A}{2}\right)$.

32. $\sin a\sin(60^\circ-a)\sin(60^\circ+a)=\frac{1}{4}\sin 3a$

33. $\cos a\cos(60^\circ-a)\cos(60^\circ+a)=\frac{1}{4}\cos 3a$.

34. $\cot a+\cot(60^\circ+a)-\cot(60^\circ-a)=3\cot 3a$.

35. $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ=\frac{1}{16}$.

36. $\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ=\frac{3}{16}$.

37. $\cos 4a=1-8\cos^2 a+8\cos^4 a$.

38. $\sin 4A=4\sin A\cos^3 A-4\cos A\sin^3 A$.

39. $\cos 6a=32\cos^6 a-48\cos^4 a+18\cos^2 a-1$.

40. $\tan 3A\tan 2A\tan A=\tan 3A-\tan 2A-\tan A$.

41. $\frac{2\cos 2^n\theta+1}{2\cos\theta+1}=(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 2^2\theta-1)$

$\ldots\ldots(2\cos 2^{n-1}\theta-1)$.

Prove that

4. $\dfrac{\sin 2A}{1+\cos 2A}=\tan A.$

5. $\dfrac{\sin 2A}{1-\cos 2A}=\cot A.$

6. $\dfrac{1-\cos 2A}{1+\cos 2A}=\tan^2 A.$

7. $\tan A+\cot A=2 \operatorname{cosec} 2A.$

8. $\tan A-\cot A=-2 \cot 2A.$

9. $\operatorname{cosec} 2A+\cot 2A=\cot A.$

10. $\dfrac{1-\cos A+\cos B-\cos (A+B)}{1+\cos A-\cos B-\cos (A+B)}=\tan \dfrac{A}{2} \cot \dfrac{B}{2}.$

11. $\dfrac{\cos A}{1 \mp \sin A}=\tan \left(45^\circ \pm \dfrac{A}{2}\right).$

12. $\dfrac{\sec 8A-1}{\sec 4A-1}=\dfrac{\tan 8A}{\tan 2A}.$

13. $\dfrac{1+\tan^2 (45^\circ-A)}{1-\tan^2 (45^\circ-A)}=\operatorname{cosec} 2A.$

14. $\dfrac{\sin \alpha+\sin \beta}{\sin \alpha-\sin \beta}=\dfrac{\tan \dfrac{\alpha+\beta}{2}}{\tan \dfrac{\alpha-\beta}{2}}.$

15. $\dfrac{\sin^2 A-\sin^2 B}{\sin A \cos A-\sin B \cos B}=\tan (A+B).$

16. $\tan \left(\dfrac{\pi}{4}+\theta\right)-\tan \left(\dfrac{\pi}{4}-\theta\right)=2 \tan 2\theta.$

17. $\dfrac{\cos A+\sin A}{\cos A-\sin A}-\dfrac{\cos A-\sin A}{\cos A+\sin A}=2 \tan 2A.$

18. $\cot (A+15^\circ)-\tan (A-15^\circ)=\dfrac{4 \cos 2A}{1+2 \sin 2A}.$

19. $\dfrac{\sin \theta+\sin 2\theta}{1+\cos \theta+\cos 2\theta}=\tan \theta.$

20. $\dfrac{1+\sin \theta-\cos \theta}{1+\sin \theta+\cos \theta}=\tan \dfrac{\theta}{2}.$

21. $\dfrac{\sin (n+1) A-\sin (n-1) A}{\cos (n+1) A+2 \cos nA+\cos (n-1) A}=\tan \dfrac{A}{2}.$

22. $\dfrac{\sin (n+1) A+2 \sin nA+\sin (n-1) A}{\cos (n-1) A-\cos (n+1) A}=\cot \dfrac{A}{2}.$

23. $\sin (2n+1) A \sin A=\sin^2 (n+1) A-\sin^2 nA.$

24. $\dfrac{\sin (A+3B)+\sin (3A+B)}{\sin 2A+\sin 2B}=2 \cos (A+B)$

8. If $\sin\theta+\sin\phi=a$ and $\cos\theta+\cos\phi=b$, find the value of $\tan\frac{\theta-\phi}{2}$.

Prove that

9. $(\cos\alpha+\cos\beta)^2+(\sin\alpha-\sin\beta)^2=4\cos^2\frac{\alpha+\beta}{2}$.

10. $(\cos\alpha+\cos\beta)^2+(\sin\alpha+\sin\beta)^2=4\cos^2\frac{\alpha-\beta}{2}$.

11. $(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2=4\sin^2\frac{\alpha-\beta}{2}$.

12. $\sin A=\dfrac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$.

13. $\cos A=\dfrac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$.

14. $\sec\left(\frac{\pi}{4}+\theta\right)\sec\left(\frac{\pi}{4}-\theta\right)=2\sec 2\theta$.

15. $\tan\left(45^\circ+\frac{A}{2}\right)=\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$.

16. $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)=\frac{1}{\sqrt{2}}\sin A$.

17. $\cos^2\alpha+\cos^2(\alpha+120^\circ)+\cos^2(\alpha-120^\circ)=\frac{3}{2}$.

18. $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$.

19. $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}=\frac{3}{2}$.

20. $\cos 2\theta\cos 2\phi+\sin^2(\theta-\phi)-\sin^2(\theta+\phi)=\cos(2\theta+2\phi)$.

21. $(\tan 4A+\tan 2A)(1-\tan^2 3A\tan^2 A)=2\tan 3A\sec^2 A$.

22. $\left(1+\tan\frac{a}{2}-\sec\frac{a}{2}\right)\left(1+\tan\frac{a}{2}+\sec\frac{a}{2}\right)=\sin a\sec^2\frac{a}{2}$.

Find the proper signs to be applied to the radicals in the three following formulae.

23. $2\cos\frac{A}{2}=\pm\sqrt{1-\sin A}\pm\sqrt{1+\sin A}$, when $\frac{A}{2}=278^\circ$.

24. $2\sin\frac{A}{2}=\pm\sqrt{1-\sin A}\pm\sqrt{1+\sin A}$, when $\frac{A}{2}=\frac{19\pi}{11}$.

25. $2\cos\frac{A}{2} = \pm\sqrt{1-\sin A} \pm \sqrt{1+\sin A}$, when $\frac{A}{2} = -140°$.

26. If $A = 340°$, prove that

$$2\sin\frac{A}{2} = -\sqrt{1+\sin A} + \sqrt{1-\sin A},$$

and

$$2\cos\frac{A}{2} = -\sqrt{1+\sin A} - \sqrt{1-\sin A}.$$

27. If $A = 460°$, prove that

$$2\cos\frac{A}{2} = -\sqrt{1+\sin A} + \sqrt{1-\sin A}.$$

28. If $A = 580°$, prove that

$$2\sin\frac{A}{2} = -\sqrt{1+\sin A} - \sqrt{1-\sin A}.$$

29. Within what respective limits must $\frac{A}{2}$ lie when

(1) $2\sin\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$,

(2) $2\sin\frac{A}{2} = -\sqrt{1+\sin A} + \sqrt{1-\sin A}$,

(3) $2\sin\frac{A}{2} = +\sqrt{1+\sin A} - \sqrt{1-\sin A}$,

and (4) $2\cos\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$?

30. In the formula

$$2\cos\frac{A}{2} = \pm\sqrt{1+\sin A} \pm \sqrt{1-\sin A},$$

find within what limits $\frac{A}{2}$ must lie when

(1) the two positive signs are taken,
(2) the two negative " " "
and (3) the first sign is negative and the second positive.

31. Prove that the sine is algebraically less than the cosine for any angle between $2n\pi - \frac{3\pi}{4}$ and $2n\pi + \frac{\pi}{4}$, where n is any integer.