$$\frac{z+1}{(z^2+1)^2} = \frac{A}{z-i} + \frac{\bar{A}}{z+i} + \frac{C}{(z-i)^2} + \frac{\bar{C}}{(z+i)^2}$$

$$\therefore\ z+1 = A(z+i)^2(z-i) + \bar{A}(z-i)^2(z+i) + C(z+i)^2 + \bar{C}(z-i)^2.$$

$$z=i:\quad 1+i = (2i)^2 C \quad \therefore\ C = -\frac{1+i}{4}$$

$$\therefore\ z+1 = A(z+i)^2(z-i) + \bar{A}(z-i)^2(z+i) - \frac{1+i}{4}(z+i)^2 - \frac{1-i}{4}(z-i)^2$$

$$\therefore\ z+1 + \frac{1+i}{4}(z+i)^2 + \frac{1-i}{4}(z-i)^2 = A(z+i)^2(z-i) + \bar{A}(z-i)^2(z+i)$$

$$\frac{1+i}{4}(z+i)^2 = \frac{1+i}{4}(z^2-1+2iz)$$

$$= \frac{z^2-1-2z}{4} + i\,\frac{z^2-1+2z}{4}.$$

$$\frac{1-i}{4}(z-i)^2 = \frac{z^2-1-2z}{4} - i\,\frac{z^2-1+2z}{4}.$$

$$\therefore\ z+1 + \frac{1+i}{4}(z+i)^2 + \frac{1-i}{4}(z-i)^2$$

$$= z+1 + \frac{z^2-1-2z}{2} = \frac{2z+2+z^2-1-2z}{2}$$

$$= \frac{1}{2}(z^2+1) = \frac{1}{2}(z-i)(z+i).$$

$$\therefore\ \frac{1}{2}(z-i)(z+i) = A(z+i)^2(z-i) + \bar{A}(z-i)^2(z+i)$$

$$\therefore\ \frac{1}{2} = A(z+i) + \bar{A}(z-i)$$

$$z=i:\quad \frac{1}{2} = 2iA \quad \therefore\ A = -\frac{i}{4},\quad \bar{A} = \frac{i}{4}.$$

$$\therefore\ \frac{z+1}{(z^2+1)^2} = \frac{i}{4}\cdot\frac{1}{z+i} - \frac{i}{4}\cdot\frac{1}{z-i} - \frac{1+i}{4}\cdot\frac{1}{(z-i)^2} - \frac{1-i}{4}\cdot\frac{1}{(z+i)^2}$$