

Number System:

The Babylonian number system is sexagesimal or base sixty, i.e. numerals range from 1 to 59. However, the numerals are expressed in a base ten system. In fact, the Babylonian system represents each sexagesimal numeral decimally, e.g. Υ is a unit, and \leftarrow is a decad, so that



is twenty-five. Numerals are usually separated by a small space. To represent



Babylonian numbers we can use commas to separate Arabic numerals, e.g. $\leftarrow \leftarrow \Upsilon \Upsilon \Upsilon$ could be $10,25 = 10 \cdot 60 + 25 = 625$. However, be warned, since early Cuneiform does not have place markers, this could be 10,25 or decimal $10,0,25 = 36025$, and many other possibilities. The Babylonians came in the Hellenistic Age to invent a zero place marker, which is the ancestor of our 0.

As a convention, we mark the separation between whole sexagesimal numbers and fractional parts with a semicolon. For example, 24;20 (which would merely appear in a text as 24,20) is $24 + 20/60 = 24 \frac{1}{3}$. $2,32;20,15 = 2 \cdot 60 + 32 + 20/60 + 15/3600 = 152 \frac{1215}{3600} = 152 \frac{27}{80}$.

In calculating, a sexagesimal number system has certain advantages, namely that $1/2$, $1/3$, $1/5$ can all be represented without recourse to repeating digits. Because of this, these numbers and their multiple combinations were preferred in educational texts and are called by modern scholars "nice" numbers. Thus we find multiplication tables for all numbers and inverse tables for nice numbers.

Example of a Problem:

The superscript numbers at the beginning are line numbers. All numbers in the texts are sexagesimal. In accordance with modern convention, semi-colons separate the whole number from the sexagesimal part and comas separate each sexagesimal numeral.

YBC 4663 (Rev. 1-12) problem 7 (Neugebauer and Sachs, 70-71)

- 1 9 (gín) is the (total expenses in) silver of a ki-lá (an excavation); I added the length and the width, and (the result is) 6;30 (GAR); $\frac{1}{2}$ GAR is [its depth],
- 2 10 gín (volume) the assignment, 6 se (silver) the wages. What are the length (and) its width?
- 3 When you perform (the operations), take the reciprocal of the wages,
- 4 multiply by 9 gín, the (total expenses in) silver, (and) you will get 4,30;
- 5 multiply 4,30 by the assignment, (and) you will get 45;
- 6 take the reciprocal of its depth, multiply by 45, (and) you will get 7;30;
- 7 halve the length and the width which I added together, (and) you will get 3;15;
- 8 square 3;15, (and) you will get 10;33,45;
- 9 subtract 7;30 from 10;33,45, and
- 10 you will get 3;3,45; take its square root, (and)
- 11 you will get 1;45; add it to the one, subtract it from the other, (and)
- 12 you will get the length (and) the width. 5 (GAR) is the length; $\frac{1}{2}$ GAR is the width.

This calculation involves a solution to a quadratic equation. What formula would you identify with the procedure?

Hints: The assignment in line 2 is the amount of volume which the worker is expected to remove for his 6 se of wages. You will have to convert se to gín at line 4. 1 gín = 3,0 se. You will have to convert gín to SAR at line 5, since the value is 45 SAR. 1,0 gín = 1 SAR. Finally, you must convert GAR measure of the depth to kùs (cubit) at line 6, since 1 SAR is 1 kùs x 1 GAR x 1 GAR. 12 kùs = 1 GAR. Incidentally, a kùs is about 20".

Solution $s = 9$ gín (expenses), $l + w = 6;30$ GAR (length + width), $d = 1/2$ GAR (depth), $a = 10$ gín (volume excavated per worker) and $p = 6$ se (wages).

$$V = 1/p * s * a = 1/6 \text{ se} * 9 \text{ gín} * 10 \text{ gín} = 5 \text{ gín} * 9 \text{ gín} * 10 \text{ gín} = 7;30 \text{ gín}^3 = 45 \text{ SAR}.$$

$$lw = \frac{V}{d} = 45 \text{ SAR} / 1/2 \text{ GAR} = 1;30 \text{ SAR/GAR} = 1;30 \text{ SAR} / 12 \text{ kùs} = 7;30 \text{ GAR}^2$$

Hence l and w equal. $\frac{l+w}{2} \pm \sqrt{\left(\frac{l+w}{2}\right)^2 - lw}$

British Museum 85196 prob. 9 Vs. II 7-16 (Neugebauer, MKT ii 44, 47-8, 53)

The following problem is important for the evidence it presents of Babylonian geometry.

- 7 A beam (?). 0;30 <GAR> (i.e., 1) gi of ...[missing text]
- 8 From the top it comes down 0;6. From [under what did it leave?]
- 9 You square 0;30. You see 0;15. Take away 0;6 from 0;30. You see 0;24.
- 10 Square 0;24. You see 0;9,36. Take away 0;9,36 from 0;15.
- 11 You see 0;5,24. What is the square-root of 0;5,24? 0,18 is the square-root. 0;18
- 12 is left on the ground. If 0;18 is left on the ground,
- 13 what comes down from the top? Square 0;18. You see 0;5,24.
- 14 Take 0;5,24 from 0;15. You see 0;9,36. 0;9,36 has
- 15 what square-root? 0;24 is the square-root. Take 0;24 from 0;30.
- 16 You see 0;6. This is what comes down. Thus is the procedure.

The figure seems to be a beam leaning against a wall. The beam is ;30 GAR or 1 gi in length. By leaning the beam on the wall, the height is reduced by 0;6 (the top of the beam is ;30 GAR - 0;6 GAR off the ground). How long is the distance of the base of the beam from the wall?

Let l = length of the beam, r = reduction of height of beam by leaning it, h = the height of the top of the beam, a = distance of the base from the wall.

The first procedure is (lines 8-12):

- Step 1: take l^2
- Step 2: take $h = l - r$
- Step 3: take h^2
- Step 4: $b = \sqrt{l^2 - h^2}$.

The second, reverse procedure is (lines 12-16):

- Step 1: take b^2
- Step 2: take $l^2 - b^2$
- Step 3: take $h = \sqrt{l^2 - b^2}$.
- Step 4: $r = l - h$.

It should be clear that this text uses a "Pythagorean Procedure."

Plimpton 322

One of the most sophisticated pieces of Babylonian mathematics is Plimpton 322 (1900-1600 B.C.E.). A transcription and decimal translation is on a separate sheet. Is it a study of Pythagorean triples (numbers such that $l^2 + b^2 = d^2$)? Or is it a method for constructing Diophantine quadratic problems (quadratic problems with rational solutions)?