Chapter 7 Solutions

P7.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90• to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.3 METHOD ONE

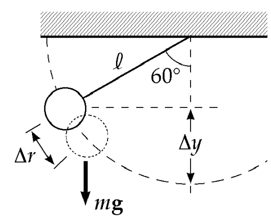

FIG. P7.3

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$.
The work done by the gravitational force on Batman is

$$W = \int_i^f F\cos\theta\, dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\, d\phi = (-80\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times 10^3\text{ J}}$$

METHOD TWO

The force of gravity on Batman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is €12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

*P7.4 Yes. Object 1 exerts some forward force on object 2 as they move through the same displacement. By Newton's third law, object 2 exerts an equal-size force in the opposite direction on object 1. In $W = F\Delta r\cos\theta$, the factors F and Δr are the same, and θ differs by 180°, so object 2 does –15.0 J of work on object 1. The energy transfer is 15 J from object 1 to object 2, which can be counted as a change in energy of €15 J for object 1 and a change in energy of +15 J for object 2.

P7.6 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$

P7.7 (a) $W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = F_x x + F_y y = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m} = \boxed{16.0\text{ J}}$

(b) $\theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}}{F\Delta r}\right) = \cos^{-1}\dfrac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$

P7.9 (a) $\vec{\mathbf{A}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$ $\qquad \theta = \cos^{-1}\dfrac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{A}} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$ $\qquad \cos\theta = \dfrac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$ $\qquad \theta = \boxed{156°}$

(c) $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$ $\qquad \theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00} \cdot \sqrt{25.0}}\right) = \boxed{82.3°}$

*P7.11 Let θ represent the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Turning by $25°$ makes the dot product larger, so the angle between $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}}$ must be smaller. We call it $\theta - 25°$. Then we have

$A\,5\cos\theta = 30$ and $A\,5\cos(\theta - 25°) = 35$

Then $A\cos\theta = 6$ and $A(\cos\theta\cos 25° + \sin\theta\sin 25°) = 7$

Dividing, $\cos 25° + \tan\theta\sin 25° = 7/6$ $\qquad \tan\theta = (7/6 - \cos 25°)/\sin 25° = 0.616$

$\theta = 31.6°$. Then the direction angle of A is $60° - 31.6° = 28.4°$

Substituting back, $A\cos 31.6° = 6$ so $\vec{\mathbf{A}} = \boxed{7.05\text{ m at }28.4°}$

P7.13 $W = \int_i^f F dx =$ area under curve from x_i to x_f

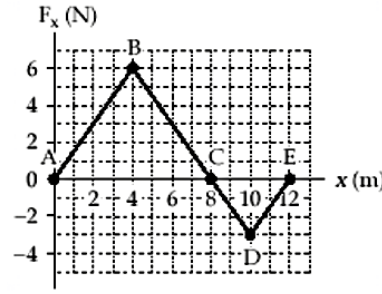

FIG. P7.13

(a) $x_i = 0$ $\quad x_f = 8.00$ m

$W =$ area of triangle $ABC = \left(\frac{1}{2}\right) AC \times$ altitude,

$W_{0\to 8} = \left(\frac{1}{2}\right) \times 8.00\text{ m} \times 6.00\text{ N} = \boxed{24.0\text{ J}}$

(b) $x_i = 8.00$ m $\quad x_f = 10.0$ m

$W =$ area of $\Delta CDE = \left(\frac{1}{2}\right) CE \times$ altitude,

$W_{8\to 10} = \left(\frac{1}{2}\right) \times (2.00\text{ m}) \times (-3.00\text{ N}) = \boxed{-3.00\text{ J}}$

(c) $W_{0\to 10} = W_{0\to 8} + W_{8\to 10} = 24.0 + (-3.00) = \boxed{21.0\text{ J}}$

P7.15 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

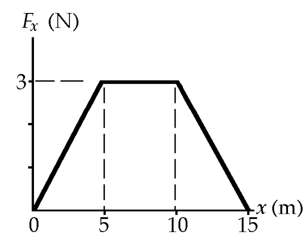

FIG. P7.15

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

P7.17 $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\text{ N}}{2.50 \times 10^{-2}\text{ m}} = 1.57 \times 10^3\text{ N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938 \text{ cm}}$

(b) Work $= \frac{1}{2} ky^2$

Work $= \frac{1}{2}(1.57 \times 10^3 \text{ N} \cdot \text{m})(4.00 \times 10^{-2} \text{ m})^2 = \boxed{1.25 \text{ J}}$

*P7.19 $\sum F_x = ma_x$: $kx = ma$

$$k = \frac{ma}{x} = \frac{(4.70 \times 10^{-3} \text{ kg})0.800(9.80 \text{ m/s}^2)}{0.500 \times 10^{-2} \text{m}} = \boxed{7.37 \text{ N/m}}$$

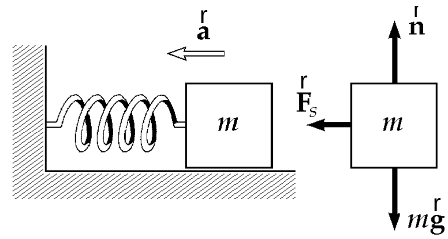

FIG. P7.19

*P7.21 Compare an initial picture of the rolling car with a final picture with both springs compressed $K_i + \sum W = K_f$. Work by both springs changes the car•s kinetic energy

$$K_i + \frac{1}{2}k_1\left(x_{1i}^2 - x_{1f}^2\right) + \frac{1}{2}k_2\left(x_{2i}^2 - x_{2f}^2\right) = K_f$$

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1600 \text{ N/m})(0.500 \text{ m})^2 + 0 - \frac{1}{2}(3400 \text{ N/m})(0.200 \text{ m})^2 = 0$$

$$\frac{1}{2}(6\,000 \text{ kg})v_i^2 - 200 \text{ J} - 68.0 \text{ J} = 0$$

$$v_i = \sqrt{\frac{2(268 \text{ J})}{6\,000 \text{ kg}}} = \boxed{0.299 \text{ m/s}}$$

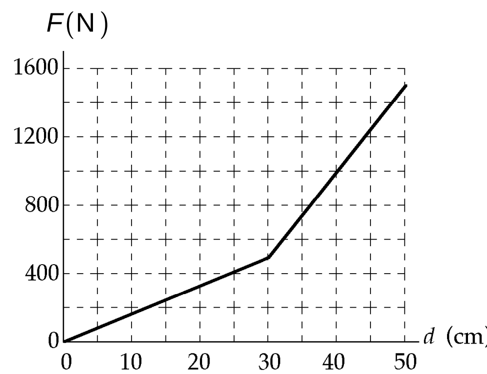

FIG. P7.21

P7.23 The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= 1.5 \text{ kg } 9.8 \text{ m/s}^2\left(\frac{1 \text{ m}}{1\,200 \text{ N}} + \frac{1 \text{ m}}{1\,800 \text{ N}}\right) = \boxed{2.04 \times 10^{-2} \text{ m}}$$

(b) We define the effective spring constant as

$$k=\frac{F}{x}=\frac{mg}{mg\left(1/k_1+1/k_2\right)}=\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1}$$

$$=\left(\frac{1\text{ m}}{1\,200\text{ N}}+\frac{1\text{ m}}{1\,800\text{ N}}\right)^{-1}=\boxed{720\text{ N/m}}$$

P7.24 See the solution to problem 7.23.

(a) $\boxed{x=mg\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}$ Both springs stretch, so the load moves down by a larger amount than it would if either spring were missing.

(b) $\boxed{k=\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1}}$ The spring constant of the series combination is less than the smaller of the two individual spring constants, to describe a less stiff system, that stretches by a larger extension for any particular load.

P7.29 (a) $K_A=\frac{1}{2}(0.600\text{ kg})(2.00\text{ m/s})^2=\boxed{1.20\text{ J}}$

(b) $\frac{1}{2}mv_B^2=K_B:\ v_B=\sqrt{\frac{2K_B}{m}}=\sqrt{\frac{(2)(7.50)}{0.600}}=\boxed{5.00\text{ m/s}}$

(c) $\sum W=\Delta K=K_B-K_A=\frac{1}{2}m\left(v_B^2-v_A^2\right)=7.50\text{ J}-1.20\text{ J}=\boxed{6.30\text{ J}}$

P7.31 $\vec{\mathbf{v}}_i=\left(6.00\hat{\mathbf{i}}-2.00\mathbf{j}\right)=\ \text{m/s}$

(a) $v_i=\sqrt{v_{ix}^2+v_{iy}^2}=\sqrt{40.0}\text{ m/s}$

$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}(3.00\text{ kg})\left(40.0\text{ m}^2/\text{s}^2\right)=\boxed{60.0\text{ J}}$

(b) $\vec{\mathbf{v}}_f=8.00\hat{\mathbf{i}}+4.00\mathbf{j}$

$v_f^2=\vec{\mathbf{v}}_f\cdot\vec{\mathbf{v}}_f=64.0+16.0=80.0\text{ m}^2/\text{s}^2$

$\Delta K=K_f-K_i=\frac{1}{2}m\left(v_f^2-v_i^2\right)=\frac{3.00}{2}(80.0)-60.0=\boxed{60.0\text{ J}}$

P7.33 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d=5.00\text{ m}$ represent the distance over which the driver falls freely, and $h=0.12\text{ m}$ the distance it moves the piling.

$\sum W=\Delta K:\quad W_{\text{gravity}}+W_{\text{beam}}=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2$

so $(mg)(h+d)\cos 0°+\left(\bar{F}\right)(d)\cos 180°=0-0$

Thus, $\bar{F}=\frac{(mg)(h+d)}{d}=\frac{(2\,100\text{ kg})\left(9.80\text{ m/s}^2\right)(5.12\text{ m})}{0.120\text{ m}}=\boxed{8.78\times10^5\text{ N}}$

The force on the pile driver is $\boxed{\text{upward}}$.

*P7.34 (a) We evaluate the kinetic energy of the cart and the work the cart would have to do to plow all the
way through the pile. If the kinetic energy is larger, the cart gets through.

K = (1/2)mv^2 = (1/2)(0.3 kg)(0.6 m/s)^2 = 0.054 J

The work done on the cart in traveling the whole distance is the net area under the graph,

W = (2 N) (0.01 m) + [(0 € 3 N)/2](0.04 m) = 0.02 J € 0.06 J = €0.04 J

$\boxed{\text{The work the cart must do is less than the original kinetic energy, so the cart does get through all the sand.}}$

(b) The work the cart does is +0.04 J, so its final kinetic energy is the remaining 0.054 J € 0.04 J = 0.014 J. Another way to say it: from the work-kinetic energy theorem,

$K_i + W = K_f$ 0.054 J € 0.04 J = 0.014 J = (1/2)(0.5 kg) v_f^2

v_f = [2(0.014 kg·m^2/s^2)/(0.3 kg)]$^{1/2}$ = $\boxed{0.306 \text{ m/s}}$

P7.37 (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}$$

When the car is at point A, the potential energy of the car-Earth system is given by

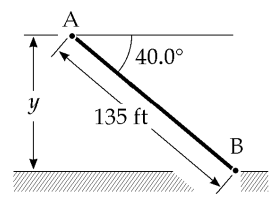

FIG. P7.37

$$U_A = mgy$$

where y is the vertical height above zero level. With $135 \text{ ft} = 41.1 \text{ m}$, this height is found as:

$$y = (41.1 \text{ m})\sin 40.0° = 26.4 \text{ m}$$

Thus,

$$U_A = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(26.4 \text{ m}) = \boxed{2.59 \times 10^5 \text{ J}}$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5 \text{ J} = \boxed{-2.59 \times 10^5 \text{ J}}$$

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) = \boxed{-2.59\times10^5 \text{ J}}$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59\times10^5 \text{ J} - 0 = \boxed{-2.59\times10^5 \text{ J}}$$

P7.38 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400 \text{ N})(2.00 \text{ m}) = \boxed{800 \text{ J}}$$

(b) From the sketch, we see that at an angle of 30.0• the child is at a vertical height of $(2.00 \text{ m})(1-\cos 30.0°)$ above the lowest point of the arc. Thus,

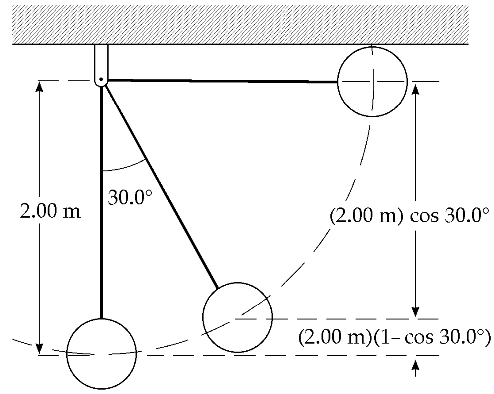

FIG. P7.38

$$U_g = mgy = (400 \text{ N})(2.00 \text{ m})(1-\cos 30.0°) = \boxed{107 \text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.