(1) Let G be a group of order pq with $p < q$ prime numbers such that $q \not\equiv 1 \mod p$. Show that G is cyclic.

(2) Let G be a group. The **commutator subgroup** of G is defined by

$$G' = \langle xyx^{-1}y^{-1} \mid x, y \in G \rangle,$$

that is, G' is the smallest subgroup of G that contains all elements of the form $xyx^{-1}y^{-1}$ with $x, y \in G$. (An important fact that you don't need for this problem is that $G' \trianglelefteq G$. See Final Exam Extra Questions.) Show the following:
   (a) $G' = \{1\}$ if and only if G is abelian.
   (b) If $H \trianglelefteq G$, then G/H is abelian if and only if $G' \leq H$.

(3) Let G be a group of order $255 = 3 \cdot 5 \cdot 17$. In class we proved that G is solvable. Follow the outline below to prove that G is, in fact, cyclic.
   (a) Show that G has a normal subgroup H of order 17.
   (b) Show that G has a normal subgroup of order 3 or of order 5.
   (c) Show that the commutator subgroup G' is contained in both H and the normal subgroup from (b), and so $G' = \{1\}$ and G is abelian. Hint: Questions 1 and 2(b).
   (d) Show G is cyclic.