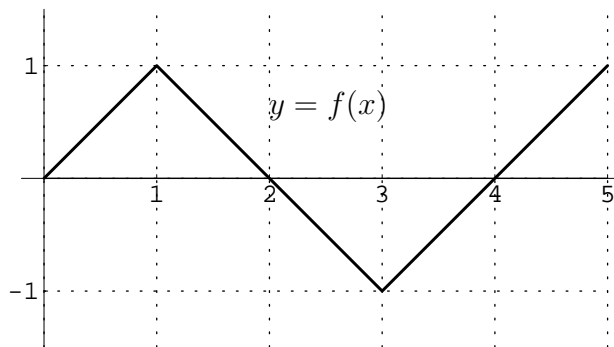


MATHEMATICS 230

MIDTERM EXAM ANSWERS

April 30, 2008

1. [5 points each] The graph of a function  $f$  is as shown.



Calculate the following:

- (a)  $\int_2^5 f(x) dx$  Answer:  $-1/2$   
 (b)  $\int_0^2 2f(x) + 1 dx$  Answer:  $4$

2. [5 points] Set up, BUT DO NOT EVALUATE, an integral which gives the area of the region of the plane bounded by the graphs of  $y = x^2$  and  $x + y = 2$ .

$$\text{Answer: } A = \int_{-2}^1 2 - x - x^2 dx$$

3. [5 points each] Calculate the following.

- (a)  $\int \frac{x}{x^2 + 1} dx$  Answer:  $\frac{1}{2} \ln(x^2 + 1) + C$   
 (b)  $\int \frac{1}{x \ln x} dx$  Answer:  $\ln(\ln x) + C$   
 (c)  $\int_0^3 \sqrt{1+x} dx$  Answer:  $14/3$   
 (d)  $\frac{d}{d\theta} \int_{\pi}^{\theta} \sin(t^2) dt$  Answer:  $\sin(\theta^2)$

4. [5 points each] Calculate the following:

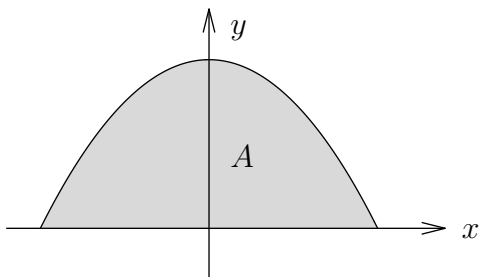
- (a)  $\int x e^{-x} dx$  Answer:  $-e^x(x + 1) + C$

(b)  $\int x e^{-x^2} dx$  Answer:  $-e^{-x^2}/2 + C$

(c)  $\int x^4 \ln x dx$  Answer:  $\frac{x^5}{25}(5 \ln x - 1) + C$

(d)  $\int \cos^3 x \sin^2 x dx$  Answer:  $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$

5. [5 points] Let  $A$  be the region bounded by the graphs of  $y = 0$ ,  $y = 1 - x^2$  as below. Set up, BUT DO NOT EVALUATE, an integral which gives the volume of the solid obtained by rotating  $A$  around the  $x$ -axis.



Answer:  $V = \int_{-1}^1 \pi(1 - x^2)^2 dx$

6. [5 points each] Convert the following to trigonometric integrals using an appropriate trigonometric substitution. DO NOT EVALUATE THE TRIG INTEGRAL!

(a)  $\int \frac{x^2}{(1 + x^2)^{3/2}} dx$  Answer:  $\int \frac{\tan^2 \theta}{\sec \theta} d\theta$

(b)  $\int x^3 \sqrt{4 - x^2} dx$  Answer:  $\int 32 \sin^3 \theta \cos^2 \theta d\theta$