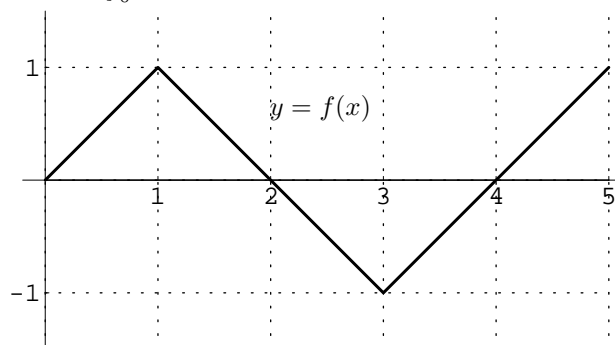


## QUIZ SOLUTIONS

## QUIZ 1

- (1) Let  $g(x) = \int_0^x f(t) dt$  where the graph of  $f$  is as shown.



Calculate the following:

- (a)  $g(3)$     **Answer:**  $1/2$   
 (b)  $g'(3)$     **Answer:**  $g'(3) = f(3) = -1$   
 (2) Calculate  $\int 3 \cos x dx$     **Answer:**  $3 \sin x + C$   
 (3) Calculate  $\int_0^2 \frac{1+x}{x} dx$

**Answer:** *Ooops, my mistake. This integral undefined:*

$$\int_0^2 \frac{1+x}{x} dx = \int_0^2 \frac{1}{x} + 1 dx = (\ln|x| + x)|_0^2 = (\ln 2 + 2) - (\ln 0 + 0).$$

*Unfortunately,  $\ln 0$  is meaningless.*

## QUIZ 2

Calculate the following integrals:

- (1)  $\int \cos \theta \sin^5 \theta d\theta$     **Answer:**  $\frac{1}{6} \sin^6 \theta + C$   
 (2)  $\int t e^{3t^2} dt$     **Answer:**  $\frac{1}{6} e^{3t^2} + C$   
 (3)  $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$     **Answer:**  $2e^{\sqrt{x}} \Big|_1^4 = 2(e^2 - e)$

## QUIZ 3

Calculate the following integrals:

- (1)  $\int \theta \sin 3\theta d\theta$     **Answer:**  $\frac{1}{9} \sin 3\theta - \frac{1}{3} \theta \cos 3\theta + C$   
 (2)  $\int x \ln x dx$     **Answer:**  $\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$   
 (3)  $\int_0^1 x^2 e^x dx$     **Answer:**  $(x^2 - 2x + 2)e^x \Big|_0^1 = e - 2$

**QUIZ 4**

Calculate the following integrals:

- (1)  $\int \cos^3 \theta \sin^3 \theta \, d\theta$     Answer:  $\frac{1}{4} \sin^4 \theta - \frac{1}{6} \sin^6 \theta + C = -\frac{1}{4} \cos^4 \theta + \frac{1}{6} \cos^6 \theta + C$
- (2)  $\int \frac{x^3}{\sqrt{16-x^2}} \, dx$     Answer:  $\frac{1}{3}(16-x^2)^{3/2} - 16\sqrt{16-x^2} + C$

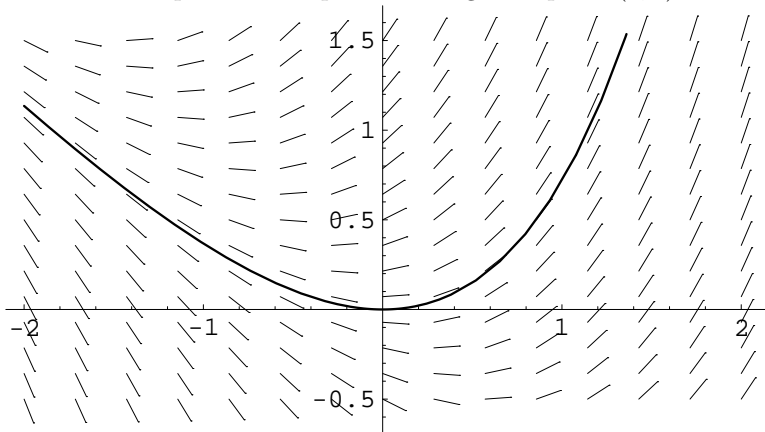
**QUIZ 5**

Let  $A$  be the region of the  $xy$ -plane bounded by the graphs of the equations  $y = \sqrt{x}$  and  $y = x$ . Find the volume of the solid of rotation obtained by rotating the region  $A$  about the  $x$ -axis.

Answer:  $V = \int_0^1 \pi(x - x^2) \, dx = \frac{\pi}{6}$

**QUIZ 6**

- (1) Below is the direction field for the differential equation  $y' = x + y$ . Sketch the graph of a solution of this equation that passes through the point  $(0, 0)$ .



- (2) Consider the differential equation  $y' = -y^2$ .  
 (a) Show that  $y = 1/(x + C)$  is a solution of this differential equation for any number  $C$ .

Answer:

$$y' = \frac{d}{dx}(x + C)^{-1} = -(x + C)^{-2} = -y^2.$$

- (b) Find a solution of the differential equation such that  $y(0) = 3$ .

Answer:  $y = 1/(x + 1/3)$

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**QUIZ 7**

- (1) Find the general solution of the differential equation  $y' = 3y/x$ . **Answer:**  $y = Ax^3$   
(2) The growth of bacteria colony is modeled by the differential equation

$$\frac{dP}{dt} = .01P \left( 1 - \frac{P}{10000} \right)$$

where  $P$  is the number of bacteria in the colony and  $t$  is the time in days. If there are 1000 bacteria now, in how many days will there be 9000?

**Answer:** *The general solution is*

$$P = \frac{10000}{1 + Ae^{-.01t}}$$

When  $t = 0$  we have  $P = 1000$ . This implies  $A = 9$ . Solving  $9000 = 10000/(1 + 9e^{-.01t})$ , we get  $e^{-.01t} = 1/81$  and so  $t = (\ln 81)/.01 \approx 440$  days.