

Chapter 16

A well grounded education: The role of perception in science and mathematics

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16.1 Introduction

One of the most important applications of grounded cognition theories is to science and mathematics education, where the primary goal is to foster knowledge and skills that are widely transportable to new situations. This presents a challenge to those grounded cognition theories that tightly tie knowledge to the specifics of a single situation. In this chapter, we develop a theory learning that is grounded in perception and interaction, yet also supports transferable knowledge. A first series of studies explores the transfer of complex systems principles across two superficially dissimilar scenarios. The results indicate that students most effectively show transfer by applying previously learned perceptual and interpretational processes to new situations. A second series shows that even when students are solving formal algebra problems, they are greatly influenced by nonsymbolic, perceptual grouping factors. We interpret both results as showing that high-level cognition that might seem to involve purely symbolic reasoning is actually driven by perceptual processes. The educational implication is that instruction in science and mathematics should involve not only teaching abstract rules and equations but also training students to perceive and interact with their world.

Scientific progress has been progressing at a dizzying pace. In contrast, natural human biology changes rather sluggishly and we are using the essentially the same kinds of brains to understand advanced modern science that have been used for millennia. Further exacerbating this tension between the different rates of scientific and neuro-evolutionary progress is that our techniques for teaching mathematics and science are not keeping up with the pace of science (Bialek and Botstein 2004). Politicians, media, and pundits have all expressed frustration with the poor state of mathematics and science education in the United States and worldwide.

There will not be any easy or singular solution to the problem of how to improve mathematics and science education. However, we believe that the consequences of improved science and mathematics education are sufficiently important¹ that it behooves

¹ Eric Hunushek, a senior fellow at the Hoover Institution at Stanford University, estimates that improving US students' maths and science grades to the levels of Western Europe within a decade would increase our gross domestic product by 4% in 2025 and by 10% by 2035.

cognitive scientists to apply their state-of-the-science techniques and results to informing the discourse on educational reform. Cognitive scientists are in a uniquely qualified position to provide expert suggestions on knowledge representation, learning, problem solving, and symbolic reasoning. These topics are core to understanding how people utilize mathematical and scientific principles. We also believe that the recent developments in embodied and grounded cognition have direct relevance to mathematics and science education, offering a promising new perspective on what we should be teaching and how students could be learning.

We will describe two separate lines of research on college students' performance on scientific and mathematical reasoning tasks. The first research line studies how students transfer scientific principles governing complex systems across superficially dissimilar domains. The second line studies how people solve algebra problems. Consistent with an embodied perspective on cognition, both lines show strong influences of perception on cognitive acts that are often associated with amodal, symbolic thought, namely cross-domain transfer and mathematical manipulation.

16.2 Transfer of complex systems principles

Scientific understanding frequently involves comprehending a system at an abstract rather than superficial level. Biology teachers want their students to understand the genetic mechanisms underlying heredity, not simply how pea plants look. Physics teachers want their students to understand fundamental laws of physics such as conservation of energy, not simply how a particular spring uncoils when weighted down (Chi *et al.* 1981). This focus on acquiring abstract principles is well justified. Science often progresses when researchers find deep principles shared by superficially dissimilar phenomena and can describe situations in terms of mathematical or formal abstractions. Finding biological laws that govern the appearance of both snails and humans (Darwin 1859), physical laws that govern both electromagnetic and gravitational acceleration (Einstein 1989), and psychological laws that underlie transfer of learning across species and stimuli (Shepard 1987) are undeniably important enterprises.

Although transcending superficial appearances to extract deep principles has inherent value, it has proven difficult to achieve (Carreher and Schliemann 2002). Considerable research suggests that, in many domains, learners do not spontaneously transfer what they have learnt, at least not to superficially dissimilar domains (Detterman 1993; Gick and Holyoak 1980, 1983). This lack of transfer based on shared deep principles has led to a major theoretical position in the learning sciences called *situated learning*. This community argues that learning takes place in specific contexts, and these contexts are essential to what is learned (Lave 1988; Lave and Wenger 1991). Traditional models of transfer are criticized as treating knowledge as a static property of an individual (Hatano and Greeno 1999), rather than as contextualized or situated, both in a real-world environment and a social community. According to situated learning theorists, one problem of traditional theorizing is that knowledge is viewed as tools for thinking that

can be transported from one situation to another because they are independent of the situation in which they are used. In fact, a person's performance on formal tasks is often worse than their performance in more familiar contexts even though, by some analyses, the same abstract tools are required (Nunes 1999; Wason and Shapiro 1971).

Providing a basis for the transfer of principles across domains is a challenge for embodied cognition approaches. Simply put, if cognition is tied to perceiving and interacting with particular scenarios, then how can we hope to have transfer from one scenario to another scenario that looks quite different (for an elaboration of this question, see Sanford, Chapter 10, this volume)? One response, given by several researchers in the situated learning community, is to give up on the prospects of transfer. The very notion of transfer is suspect in that community because of their focus on contextualized knowledge. In one often-reported study, Brazilian children who sell candy may be quite competent at using currency even though they have considerable difficulty solving word problems requiring calculations similar to the ones they use on the street (Nunes 1999). Transfer of mathematical knowledge from candy selling on the streets to formal algebra in the classroom is neither found nor expected by situated learning theorists. So, in order to understand success in the classroom, situated learning theorists place the emphasis on the context of the classroom and to understand success in the streets, the street context is studied.

We share with the situated learning community an emphasis on grounded knowledge. However, we mean something quite different by 'grounding'. For the situated learning community, knowledge is contextualized in the actions, social goals, and physical details of particular concrete scenarios such as selling candy on a Brazilian street. Situated learning theorists equate knowledge with problem solving activities that are cued, in some sense 'grounded', by the features of these concrete scenarios. Generalization is thus a problem-solving behaviour exhibited over a set of scenarios (Greeno 1997). Situated learning theorists typically criticize cognitive theorists for abstracting away from these scenarios to explain generalization. However, for us, knowledge is not simply in the extracted verbal or formal description of a situation, but rather in the perceptual interpretations and motoric interactions involving a concrete scenario. While we look to embodied experiences to ground learning, we still believe in the possibility and power of transfer across contexts. It is possible to learn principles in a grounded way that enables the principles to be recognized in the myriad of concrete forms that they can take.

Typically, cognitive theories of transfer are expressed in terms of the acquisition of abstracted formalisms that lead to the direct perception of mathematical structures and patterns. Instead, we believe that learning that transfers to new scenarios and transports across domains, most often proceeding not through acquiring and applying symbolic formalisms but rather through modifying automatically perceived similarities between scenarios by training one's perceptual interpretations. In the following two sections we will: (1) argue for the desirability of cross-domain transfer of scientific principles, and (2) show how this transfer is compatible with a perceptually grounded understanding of science.

16.2.1 Complex systems and transfer

Our claim for the desirability and possibility of transfer of scientific principles across domains is based on the power of *complex systems theories*. A complex systems perspective provides a unifying force, bringing increasingly fragmented scientific communities. Journals, conferences, and academic departmental structures are becoming increasingly specialized and myopic (Csermely, 1999). One possible response to this fragmentation of science is to simply view it as inevitable. Horgan (1996) argues that the age of fundamental scientific theorizing and discoveries has passed, and that all that is left to be done is refining the details of theories already laid down by the likes of Einstein, Darwin, and Newton.

Complex systems researchers offer an alternative to increasing specialization. They have pursued principles that apply to many scientific domains, from physics to biology to social sciences. For example, reaction–diffusion equations that explain how cheetahs develop spots can be used to account for geospatial patterns of Democrats and Republicans in America. The same abstract schema underlies both phenomena – two kinds of elements (e.g. two skin cell colours, or two political parties) both diffuse outwards to neighbouring regions but also inhibit one another. The process of *diffusion-limited aggregation* is another complex adaptive systems explanation that unifies diverse phenomena: individual elements enter a system at different points, moving randomly. If a moving element touches another element, they become attached. The emergent result is fractally connected branching aggregates that have almost identical statistical properties (Ball 1999). This process has been implicated in the growth of human lungs (Garcia-Ruiz *et al.* 1993), snowflakes (Bentley and Humphreys 1962), and cities (Batty, 2005). These examples all describe principles that can be instantiated with highly dissimilar sets of individual elements, but with interactions between the elements that are captured by very similar algorithmic rule sets (Bar-Yam 1997).

Generally speaking, complex systems are systems made up of many units (oftentimes called agents), whose simple interactions give rise to higher-order emergent behaviour. Typically, the units all obey the same simple rules, but because they interact the units that start off homogenous and undifferentiated may become specialized and individualized (O'Reilly 2001). Despite the lack of a centralized control, leader, recipe, or instruction set, these systems naturally self-organize (Resnick 1994; Resnick and Wilensky 1993). Many real-world phenomena can be explained by the formalisms of complex adaptive systems, including the foraging behaviour of ants, the development of the human nervous system, the growth of cities, growth in the internet, the perception of apparent motion, mammalian skin patterns, pine cone seed configurations, and the shape of shells (Casti 1994; Flake 1998).

In the following studies, we will focus on one particular complex systems principle, *competitive specialization*, because we have taught this principle in our own undergraduate courses, and because we have conducted controlled laboratory studies on students' appreciation and use of the principle (Goldstone and Sakamoto 2003; Goldstone and Son 2005). A well-worked out example of competitive specialization is the development of neurons in the primary visual cortex that start off homogeneous and become

specialized to respond to visually presented lines with specific spatial orientations (von der Malsburg 1973). Another application is the optimal allocation of agents to specialize to different regions in order to cover a territory. In these situations, a good solution is found if every region has an agent reasonably close to it. For example, an oil company may desire to place oil drills such that they are well spaced and cover their territory. If the oil drills are too close, they will redundantly access the same oil deposit. If the oil drills do not cover the entire territory, then some oil reserves will not be used.

Ants and food

The first example of competitive specialization involves ants foraging food resources drawn by a user. The ants follow exactly three rules of competitive specialization: at each time step, (1) a piece (pixel) of food is randomly selected from all of the food drawn by a user, (2) the ant closest to the piece moves at one rate, and (3) all of the other ants move at another rate. In interacting with the simulation, a learner can reset the ants' positions, clear the screen of food, draw new food, place new ants, move ants, start/stop the ants' movements, and set a number of simulation parameters. The two most critical user-controlled parameters determine the movement speed for the ant that is closest to the selected food (called 'closest rate' in Figure 16.1) and the movement speed for all other ants ('Not closest rate').

Starting with the initial configuration of three ants and three food piles shown in Figure 16.1, several important types of final configuration are possible and are shown in Figure 16.2. If only the closest ant moves toward a selected piece of food, then this ant will be the closest ant to *every* patch of food. This ant will continually move to new locations on every time step as different patches are sampled, but will tend to hover around the center-of-mass of the food patches. The other two ants will never move at all because they will never be the closest ant to a food patch. This configuration is suboptimal because the average distance between a food patch and the closest ant (a quantity that is continually graphed) is not as small as it would be if each of the ants specialized for a different food pile. On the other hand, if all of the ants move equally quickly, then they will quickly converge to the same screen location. This also results in a suboptimal solution because the ants do not cover the entire set of resources well – there will be patches that do not have any nearby ant. Finally, if the closest ant moves more quickly than the other ants but the other ants move too, then a nearly optimal configuration is achieved. Although one ant will initially move more quickly toward all selected food patches than the other ants, eventually it will specialize to one patch and the other ants will then be closer to the other patches, allowing for specialization.

An important, subtle aspect of this simulation is that poor patterns of resource covering are self-correcting so the ants will almost always self-organize themselves in a one-to-one relationship to the resources regardless of the lopsidedness of their original arrangement, provided good parameter values are used.

Pattern learning

The second example of competitive specialization involves sensors responding to patterns drawn by the user. Just like the ants/food scenario, the pattern-learning simulation follows the same three rules of competitive specialization. At the beginning of the

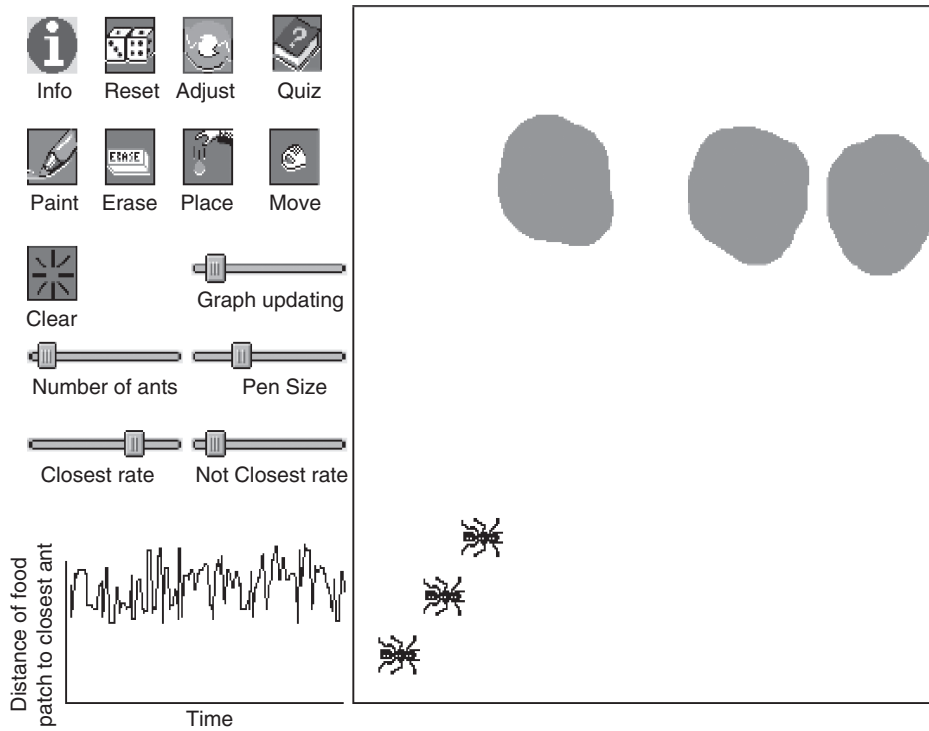


Fig. 16.1 A screen dump of an initial configuration for the 'ants and food' computer simulation. At each time step, a patch of food is randomly selected, and the ant closest to the patch moves toward the patch with one speed (specified by the slider 'Closest rate') and the other ants move toward the patch with another speed ('Not closest rate').

simulation, the sensors respond to random noise. But at each time step, a pattern is randomly selected from all of the patterns drawn by a user, and the sensor most similar to that pattern adapts to become more similar to that pattern at a particular rate. All of the other sensors adapt towards the selected pattern at another rate. Users can reset sensors to become random again, draw patterns, erase patterns, copy patterns, add noise, start/stop pattern learning, and change a number of parameters. The most important parameters are the rates of adaptation for the most similar and not most similar sensors (Figure 16.3). Note that the final configurations shown in Figure 16.2 also apply to the pattern-learning simulation.

Using these two case studies of the competitive specialization principle, we are now in a good position to state our challenge for grounded educational practices. Our desiderata is a teaching method that will promote transfer from one example of competitive specialization to another. How can students who learn about competitive specialization with the ants and food scenario spontaneously apply what they have learned to the pattern-learning situation? Given the lack of superficial perceptual features shared by

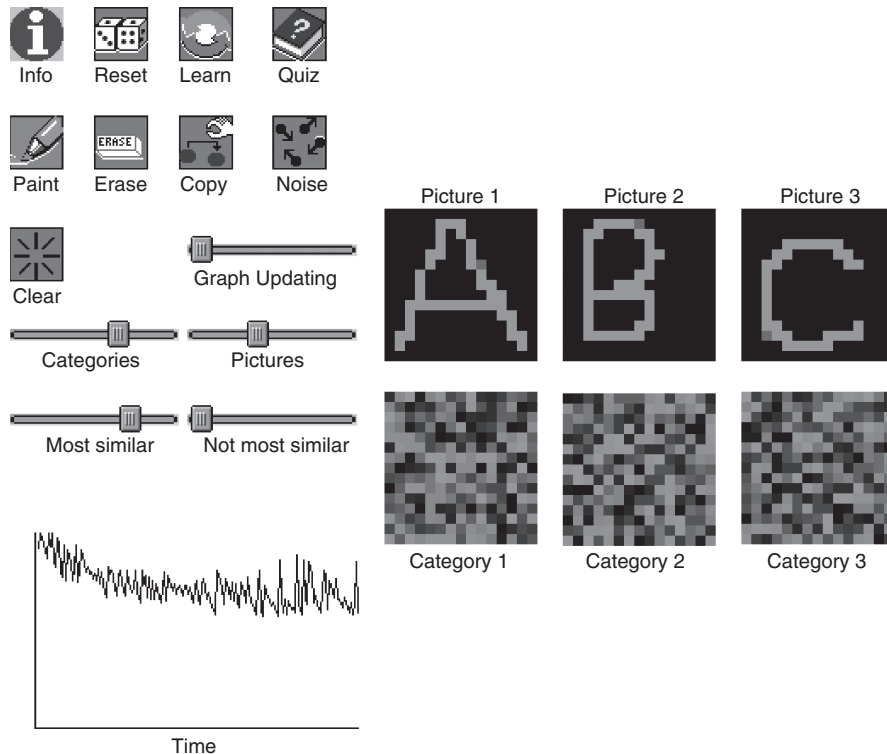


Fig. 16.2 A screen dump for the 'pattern learning' simulation. Users draw pictures, and prior to learning, a set of categories are given random appearances. During learning, a picture is selected at random, and the most similar category to the picture adapts its appearance toward the picture at one rate (specified by the slider 'Most similar') while the other categories adapt toward the picture at another rate ('Not most similar').

Figures 16.1 and 16.2, can the transfer be due to cognitive processes grounded in perception and interaction? The next section sketches out our affirmative answer.

16.2.2 How to teach complex systems principles

Powerful complex systems principles cut across scientific disciplines. We are therefore inherently interested in *transportable* knowledge – knowledge that can be applied to domains significantly beyond those originally presented. In setting the stage for our grounded approach to cognition, we will first describe a traditional alternative approach to transfer.

Transfer via formalisms

It is understandable why so many educators have been drawn to couch their textbook and classroom teaching in terms of formalisms. The formalizations established by algebra, set theory, and logic are powerful because they are domain-general. The same

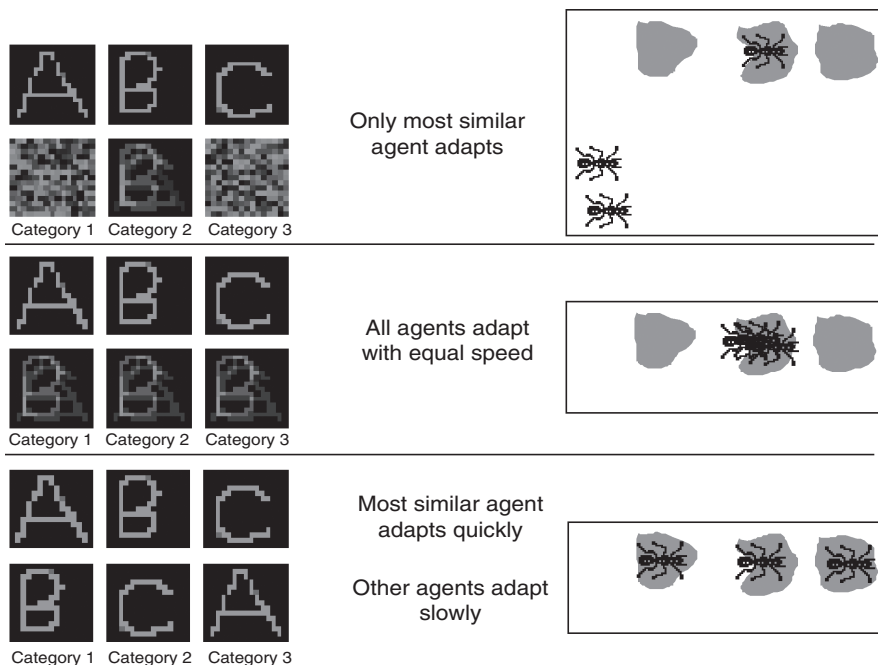


Fig. 16.3 The basis for the isomorphism between the ants-and-food and pattern-learning simulations. If only the most similar agent to a resource adapts, then often a single agent will move toward the average of all of the resources. If all agents adapt equally quickly, they will all move toward the average position. If the agent closest to a resource patch moves much faster than the other agents but all agents move a bit, then each of the agents typically becomes specialized for one resource type.

equation for probability can apply equally well to shoes, ships, sealing wax, cabbages, and kings. Not only is no ‘customization’ of equations to a content domain required, it is forbidden. The formalism sanctions certain transformations that are provably valid, and once a domain has been translated to the formalism one can be assured that the deductions drawn from the transformations will be valid. Logic and maths are the best examples that we can think of for the kind of symbolic processing that Glenberg *et al.* (Chapter 1, this volume) have in mind when they describe symbols as abstract, amodal, and manipulated by using explicit rules. This is not to imply that we believe that engaging in mathematics is a purely symbolic activity. Crucially, we do not, but it is the best candidate domain for symbolic reasoning because the rules for mathematics are relatively noncontentious compared with, for example, the rules for language. Furthermore, students receive a significant amount of training with the generic, abstract form of mathematical rules and explicitly learn how to transform symbolic representations.

Mathematical formalisms are the epitome of representations that abstract over domains. Given this, there is some reason to believe that they will promote free transfer of knowledge from one domain to another. This notion is endemic in high-school

mathematics curricula, which often feature abstract formalisms that, once presented, are only subsequently fleshed out by examples. Systematic analyses of mathematics textbooks have shown that formalisms tend to be presented before worked-out examples and that this tendency increases with grade level (Nathan *et al.* 2002).

Although formalisms may seem plausible candidates for producing transportable knowledge, the literatures from cognitive science and education give three grounds for skepticism. First, the mismatch between how mathematics is presented and how it is conceived by practitioners has often been noted (Lakoff and Nuñez 2000; Thurston, 1994; Wilensky 1991). As teachers' mathematical expertise increases, they increasingly believe that using formalisms is a requirement for solving mathematical problems, even though this kind of strong association is not found in actual students' performance (Nathan and Petrosino 2003). Hadamard has complained that the true heart of a mathematical proof, the intuitive conceptualization, is ignored in the formal description of the proof steps themselves (Hadamard 1949). The scholarly articles contain the step-by-step, formally sanctioned methods, but if one wishes to understand where the idea for these steps comes from, then one must attempt to generate the visuospatial inspiration oneself, without much insight from the published report. In mathematics textbooks, formalisms are either treated as givens, or when they are derived, they tend to be formally derived from other formalisms.

The second reason for skepticism is that formalism-based transfer can only work if people spontaneously recognize when an equation can be applied to a situation. In fact, the connection between equations and scenarios is typically indirect and difficult to see (Chi 2005; Hmelo-Silver and Pfeffer 2004; Nathan *et al.* 1992; Penner 2000). Students often have difficulty finding the right equation to fit a scenario, even when they know both the equation and the major elements of the scenario (Ross 1987, 1989).

The third reason for skepticism is that transfer potential is almost certainly not maximized by creating the most efficient minimalist representation possible, one that leaves out irrelevant content information and captures only relevant structural information. Formalisms are maximally content-independent, but this is the precise property that often leads them to be cognitively inert. They offer little by way of scaffolding for understanding, and may not generalize well because cues to resemblance between situations have been stripped away. Research has shown that the content-dependent semantics of a scenario promotes transfer, and also gives valuable clues about the kinds of formalism that are likely to be useful (Bassok 1996, 2001; Bassok *et al.* 1998).

Grounded transfer

We would like to propose an alternative, intermediate position to the extreme positions that transfer should proceed via formalisms and that remote transfer cannot be achieved at all (Detterman 1993). Our position is that transfer can be achieved, not by teaching students abstract formalisms but through the interpretation of grounded situations. One motivation for this method comes from work with students solving story problems with and without access to physical manipulatives that can be used to act out the story (Glenberg *et al.* 2004). The physical manipulatives helped students to understand and

solve the problems, and these benefits transferred to conditions in which students simply imagined using the physical objects (Glenberg *et al.*, in press). The process of interpreting actual or imagined objects was instrumental in getting students to properly understand the underlying mathematical issues involved in a situation.

Another motivation for this method comes from our observation of students learning principles of complex systems in our classes and laboratory experiments. We have observed that students often interact with our pedagogical simulations by actively interpreting the elements and their interactions. Their interpretations are grounded in the particular simulation with which they are interacting. Furthermore, because the interpretations may be highly selective, perspectival, and idealized, the same interpretation can be given to two apparently dissimilar situations. The process of interpreting physical situations can therefore provide understandings that are grounded yet transportable. Practicing what we preach, we will now ground our notion of interpretive generalization with the competitive specialization principle using an example from our 'complex adaptive systems' undergraduate course. The relevant simulations can be accessed at: <http://cognitrn.psych.indiana.edu/rgoldsto/complex/>.

Our laboratory and classroom investigations with these two demonstrations of competitive specialization have shown that students can, under some circumstances, transfer what they learn from one simulation to another (Goldstone and Sakamoto 2003; Goldstone and Son 2005). In our experiments, we first gave students a period of focused exploration with the ants-and-food simulation because it embodies competitive specialization in a relatively literal and spatial manner, then we let students explore the pattern-learning simulation. We probe their understanding of the latter simulation both through multiple-choice questions designed to measure their appreciation of the principle of competitive specialization in the pattern learning context, and through a performance-based measure of how quickly students can create parameter settings whereby categories automatically adapt so as to represent the major classes of input patterns.

Using this method, we found that students show better understanding of the pattern-learning simulation when it has been preceded by the ants-and-food simulation than by a simulation governed by a different principle. Student interviews indicate that a major cause of the positive transfer is training perceptual interpretations of grounded situations. After exposure to ants-and-food, several students spontaneously applied the same perceptual representation to pattern learning. The visuospatial dynamics of the ants-and-food simulation are aptly applied. In fact, when originally presenting their general competitive learning algorithm, Rumelhart and Zipser use a visualization along the lines of ants-and-food to give the reader a solid intuition for how their algorithm works. This visualization, shown in Figure 16.4, depicts the adaptation of categories in a high-dimensional space as the movement of those categories in three-dimensional space. Our students' spontaneous visualizations have elements in common with this professional visualization.

Typical elements in our students' visualizations are shown in Figure 16.5. A student was asked to visually describe what would happen when there are two categories and four input pictures that fell into two clusters: variants of As and variants of Bs. The student

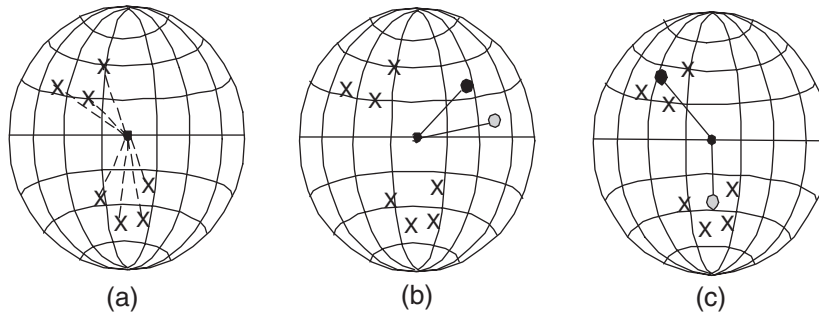


Fig. 16.4 A standard visualization of competitive specialization, adapted from an illustration by Rumelhart and Zipser (1985). Panel A shows the coordinates of seven objects, represented by 'x's, in a three-dimensional space. In panel B, the random starting coordinates for two categories are shown by shaded circles. Panel C shows the resulting coordinates for the categories after several iterations of learning by competitive specialization.

drew the illustration in panel A. In this illustration, adaptation and similarity are both represented in terms of space, much as they are in Figure 16.4. The two categories ('Cat1' and 'Cat2') are depicted as moving spatially toward the spatially-defined clusters of 'A's and 'B's, and the 'A' and 'B' pictures are represented as spatially separated. The context for panel B was a student who was asked what problem might occur if the most similar detector to a selected picture moved quickly to the picture, while the other detectors did not move at all. The student showed a single category (shown as the box with a '1') moving toward, and eventually oscillating between, the two pictures. Again, similarity is represented by proximity and adaptation by movement.

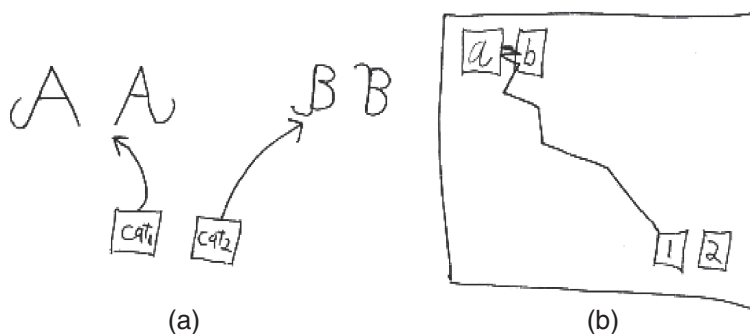


Fig. 16.5 Typical visualizations of students expressing their knowledge of pattern learning. In panel A, the student represents the adaptation of categories by moving them through space towards two clusters of stimuli. The similarity of the two variants of 'A' is represented by their spatial proximity. In panel B, a student was asked to illustrate a problem that arises when the closest category to a pattern adapts, but the others do not adapt at all: a single category is shown oscillating between two patterns. Note the similarity to the top panel of Figure 16.3 with the ants-and-food simulation.

Students' verbal descriptions also give evidence of the spatial diagrams that they use to explicate pattern learning. Students frequently talk about a category 'moving over to a clump of similar pictures.' Another student says that 'this category is being pulled in two directions, toward each of these pictures.' A third student also uses spatial language when saying, 'These two squares are close to each other, so they will tend to attract the same category to them.' In this final example, the two square pictures were not physically close to each other on the screen, they were separated by a picture of a circle.

All three of these reports show that students are understanding visual similarity in terms of spatial proximity. The squares are close in terms of their visual appearance and not their spatial distance. Categories do not change their position on the screen, but only in a high-dimensional space that describes pictures. Ants move in visual space, while categories adapt in description space. In doing so, both ants and categories are adapting so that they cover their resources (food or pictures) well. Our students frequently find making the connections between adaptation and motion, and between similarity and proximity to be natural, and much more so after they have had experience with the ants-and-food simulation.

How should we understand these connections? One common approach is to claim that ants-and-food uses space literally, while students' understanding of pattern learning treats space metaphorically or figuratively. Movement in space is viewed as an apt analogy for adaptation. The problem with describing space as figurative in pattern learning is that space is very concrete and grounded for students who draw diagrams like those shown in Figure 16.5. Our alternative proposal is that students are using the literal, spatial models that they learned while exploring ants-and-food to understand and predict behaviour in pattern learning. There is a process where visual similarity is thought of in terms of spatial proximity, but once that mapping has been made, students conduct the same kinds of mental simulations that they perform when predicting what will happen in new ants-and-food situations. So, we do not believe that students abstract a formal structural description that unifies the two simulations. Instead, they simply apply to a new domain the same perceptual routines that they have previously acquired. By one account, the domains are visually dissimilar. However, once appropriately construed, the domains are highly similar visually. In fact, students seem to use superficial spatial properties to construe the ants-and-food simulation.

An interesting aspect of both illustrations in Figure 16.5 is that although space is being used to represent similarity, there are still vestiges of space being used to represent space. In these, and other, student illustrations, the categories are placed below the input pictures, just as they are in the interface shown in Figure 16.2. Even though students have difficulty explicitly describing the general principle that governs both simulations, the perceptual trace left by ants-and-food can prime an effective perceptual construal of pattern learning.

What is most striking about the students' descriptions of pattern learning is the extent of knowledge-driven perceptual interpretation. These interpretations are not simply formalisms grafted onto situations. Rather, the interpretations affect the perception of

the simulation elements. Students see single categories trying to ‘cover’ multiple pictures, pictures being ‘close’ or ‘far’ in terms of their appearances, and categories ‘moving’ toward pictures. To the experienced eye, identical perceptual configurations are visible in the two simulations. Figure 16.3 shows equivalent configurations. The critical point is that these pairs are obviously not perceptually similar under all construals. It is only to the student who has understood the principle of competitive specialization as applied to ants-and-food that the two situations appear perceptually similar. One moral is: just because a reminding is perceptually-based does not make it necessarily superficial. A superficial construal of the left and right columns of Figure 16.3 would not reveal much commonality. However, a dynamic, visual, and spatial construal of pattern learning can be formed that makes the solutions of ants-and-food directly relevant.

In opposition to the traditional schism between perceptual and conceptual processes, our work on transferring complex systems principles suggests that the perceptual interpretation process is key to generating transportable conceptual understandings. Perceptual interpretation requires both a physically present situation and construals of the elements of the situation and their interactions. Simply giving the interpretation is not adequate. Like equations, standalone descriptions are unlikely to foster transfer because of their lack of contact to applicable situations. When we simply give students the rules of pattern learning, they are seldom reminded of ants-and-food. Physically grounding a description is one of the most effective ways of assuring that it is conceptually meaningful. However, giving the grounded situation but no interpretation is inadequate. It is only when the interpretation is added to the presentation of two superficially dissimilar situations that the resemblance becomes apparent. Students are not just seeing events, they are seeing events as instantiating principles. This act of interpretation, an act of ‘seeing something as X’ rather than simply seeing it (Wittgenstein 1953), is the key to cultivating transportable knowledge.

16.3 Perceptual learning

An important plank of our proposal is that the similarity between situations governed by the same complex systems principle can be used to promote transfer even if the situations are dissimilar to the untutored eye, and even if the similarity is not explicitly noticed. This claim apparently contradicts the empirical evidence for very limited transfer between remote situations (Detterman 1993; Reed *et al.* 1974, but see also Barnett and Ceci 2002 for a balanced evaluation of the evidence). Our claim is that the perceived similarity of situations is malleable, not fixed by objective properties of the situations themselves. It may well be that remotely related situations rarely facilitate each other. However, well designed activities can alter the perceived similarity of situations, and what were once dissimilar situations can become similar to one another with learning (Goldstone 1998; Goldstone and Barsalou 1998). Our hope, then, is not to have students transfer by connecting remotely related situations, but rather to have students warp their psychological space so that formerly remote situations become similar (Harnad *et al.* 1995).

There is already good evidence for this kind of warping of perception due to experience and task requirements (Goldstone 1994; Goldstone *et al.* 2001; Livingston *et al.* 1998; Özgen 2004; Roberson *et al.* 2000).

Graphical, interactive computer simulations (Goldstone and Son 2005; Jacobson 2001; Jacobson and Wilensky 2005; Resnick 1994; Wilensky and Reisman 1999, 2006) offer attractive opportunities for promoting generalization. Principles are not couched in equations, but rather in dynamic interactions among elements (Nathan, Chapter 18, this volume; Nathan *et al.* 1992). Students who interact with the simulations actively interpret the resulting patterns, particularly if guided by goals abetted by knowledge of the principle. Their interpretations are grounded in the particular simulation, but once a student has practiced building an interpretation, it is more likely to be used for future situations. In contrast to explicit equation-based transfer, perceptually-based priming is automatic. For example, an ambiguous man/rat drawing is automatically interpreted as a man when preceded by a man and as a rat when preceded by a rat (Leeper 1935). This phenomenon, replicated in countless subsequent experiments on priming (see Goldstone 2003 for a theoretical integration), is not ordinarily thought of as transfer, but it is an example of a powerful influence on perception due to prior experiences. This kind of automatic shift of perceptual interpretation accompanies engaged interaction with complex system simulations. In these cases, generalization arises, not just from the explicit and effortful application of abstract formalisms, but critically from the simple act of ‘rigging up’ a perceptual system to interpret a situation according to a principle, and leaving this rigging in place for subsequently encountered situations.

16.3.1 Perception and idealization

In arguing for an embodied basis for transfer complex systems principles, it is important to clarify what we mean to entail by ‘embodied’. To us, embodiment is compatible with idealization. Following Glenberg *et al.* (Chapter 1, this volume), we consider our students’ understanding of complex systems principles to be embodied when it depends on activity in systems used for perception and action. Both our computer simulations, and the mental simulations of those simulations are perceptual in that they incorporate spatial and temporal information, and presumably do so by using brain regions that are dedicated to perceptual processing. Arnheim (1970), Barsalou (1999; Barsalou *et al.* 2003; Simmons and Barsalou 2003), Glenberg (Chapter 2, this volume; Glenberg *et al.*, in press), Schwartz (Schwartz and Black 1999), Roy (Chapter 11, this volume) and others have argued that our concepts are not amodal and abstract symbolic representations, but rather are grounded in the external world via our perceptual systems.

Embodied knowledge has a major advantage over amodal representations – they preserve aspects of the external world in a direct manner so that the mental simulations and the simulated world automatically stay coordinated even without explicit machinery to assure correspondence. Dimensions in the model naturally correspond to dimensions of the modelled world. Given this characterization, it is clear that embodied representations neither need to superficially resemble the modelled world nor preserve all of the

raw, detailed information of that world (Barsalou 1999; Shepard 1984). Our experience with students' understanding of complex systems computer simulations indicates that simulations lead to the best transfer when they are relatively idealized. Goldstone and Sakamoto (2003) gave students experience with two simulations exemplifying the principle of competitive specialization – ants-and-food, followed by pattern learning. Figure 16.6 shows the design for the experiments. Ants-and-food was either presented using line drawings of ants and food, or simplified geometric forms. Overall, students showed greater transfer to the second scenario when the elements were graphically idealized rather than realistic. Interestingly, the benefit of idealized graphical elements was largest for our students who had relatively poor understanding of the initial simulation. It might be thought that strong contextualization and realism would be of benefit to those students with weak comprehension of the abstract principle. Instead, it seems that these poor comprehenders are particularly at risk for interpreting situations at a superficial level, and using realistic elements only encourages this tendency. Smith (2003) has

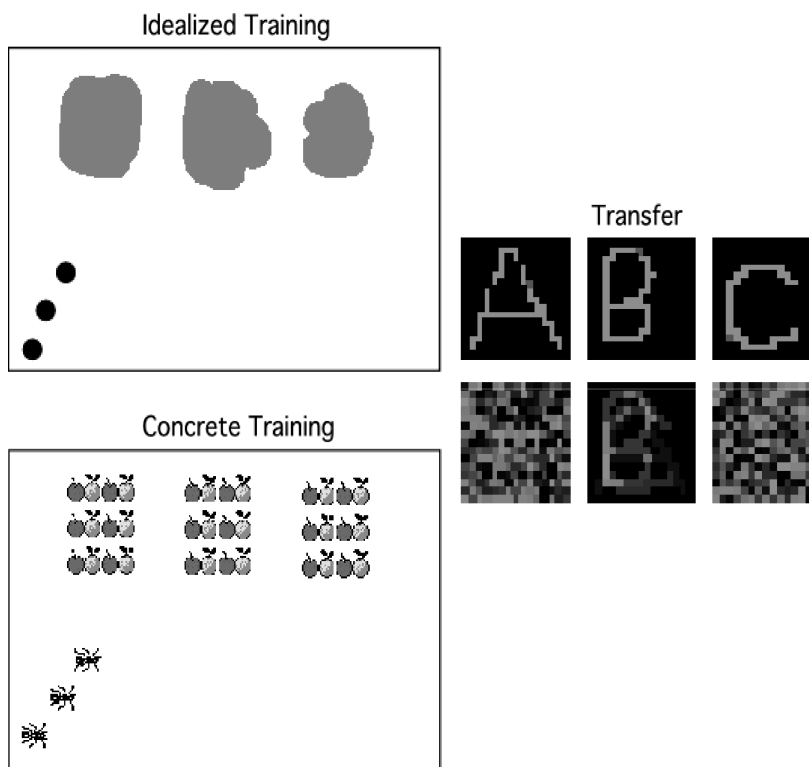


Fig. 16.6 The design of an experiment how concrete visual simulations should be to promote transfer (Goldstone and Sakamoto, 2003). Better transfer performance was found for the idealized, relative to concrete, training condition, particularly for students with poor comprehension of the training simulation.

found consistent results with rich versus simple geometric objects, and Sloutsky *et al.* (2005) have found similar benefits of idealized over concrete symbols for learning about algebraic groups (see also Wiley 2003 for a review of the benefits and hazards of concretization).

Judy DeLoache's *model room paradigm* (DeLoache 1991, 1995; DeLoache and Burns 1994; DeLoache and Marzolf 1992) has also found that idealization can promote symbolic understanding. A child around the age of 2.5 years is shown a model of a room and watches as a miniature toy is hidden behind or under a miniature item of furniture in the model. The child is told that a larger version of the toy is hidden at the corresponding piece of furniture in the room. Children were better able to use the model to find the toy in the actual room when the model was a two-dimensional picture rather than a three-dimensional scale model (DeLoache 1991; DeLoache and Marzolf 1992). Placing the scale model behind a window also allowed children to more effectively use it as a model (DeLoache 2000). DeLoache and colleagues (DeLoache 1995; Uttal *et al.* 1997, 1999) explain these results in terms of the difficulty in understanding an object as both a concrete, physical thing and as a symbol standing for something else.

A natural question to ask is, 'How can we tell whether a particular perceptual detail will be beneficial because it provides grounding or detrimental because it distracts students from appreciating the underlying principle?' The answer depends on the nature of complex systems principles, the learner's cognitive development, and what is easily implemented in the mental simulations. Complex systems models are typically characterized by simple, similarly configured elements that each follow the same rules of interaction. For this reason, idiosyncratic element details can often be eliminated, and information about any element only need be included to the extent that it affects its interactions with other elements. Mental simulations are efficient at representing spatial and temporal information, but are highly capacity-limited (Hegarty 2004a). Under the assumption that a student's mental model will be shaped by the computational model that informs it, the following prescriptions are suggested for building computer simulations of complex systems: (1) eliminate irrelevant variation in elements' appearances, (2) incorporate spatial-temporal properties, (3) do not incorporate realism just because it is technologically possible, (4) strive to make the element interactions visually salient, and (5) be sensitive to peoples' capacity limits in tracking several rich, multifaceted objects.

Another empirically supported suggestion for compromising between grounded and idealized presentations is to begin with relatively rich, detailed representations, and gradually idealize them over time (Goldstone and Son 2005). This regime of 'concreteness fading' was proposed as a promising pedagogical method because it allows simulation elements to be both intuitively connected to their intended interpretations, but also eventually freed from their initial context in a manner that promotes transfer.

16.3.2 Perceptual grounding in mathematics

In the previous section, we contrasted the benefits of presenting scientific principles using perceptually grounded and interactive simulations, rather than traditional

algebraic equations. One reason why complex system computer simulations are so pedagogically effective is that they mesh well with human mental models (Gentner and Stevens 1983; Graesser and Jackson, Chapter 3, this volume; Graesser and Olde 2003; Hegarty 2004b). These computer models enact the same kind of step-by-step simulation of elements and interactions that effective mental models do. However, even though we advocate the perceptual grounding provided by simulations for pedagogical use, our position is not that algebraic equations are ungrounded or that they somehow transcend perception. In fact, some of our recent experiments indicate surprisingly strong influences of apparently superficial perceptual grouping factors on people's algebraic reasoning.

To study the influence of perceptual grouping on mathematics, we gave undergraduate participants a task to judge whether an algebraic equality was necessarily true (Landy and Goldstone, under review). The equalities were designed to test their ability to apply the order of precedence of operations rules. Our participants have learned the rule that multiplication precedes addition. Our instructions and post-experiment interviews confirm that participants know this rule. However, we were interested in whether perceptual and form-based groupings would be able to override their general knowledge of the order of precedence rules. We tested this by having grouping factors either consistent or inconsistent with order of precedence. For example, if shown the stimuli in the top row of Figure 16.7, participants would be asked to judge whether $R * E + L * W$ is necessarily equal to $L * W + R * E$. These terms are necessarily equal, and so the correct response would be 'yes' for this trial. On some trials, the physical spacing between the operators was consistent with the order of precedence – large spacings separated the terms related by addition and small spacings separated terms related by multiplication. A powerful principle of Gestalt perception (Koffka 1935) is that close objects are seen as grouping together. Accordingly, small spaces between terms to be multiplied together would be expected to facilitate grouping them together early, consistent with the order of precedence. On other trials, the physical spacing was inconsistent with the order of precedence, with larger spaces around '*'s than '+'s.

The influence of spacing was profound, as shown in Figure 16.8. When physical spacing was inconsistent with order of precedence rules, six times as many errors were made relative to when the spacing was consistent. However, for problem types where the order of operations did not influence the validity of the equation ('insensitive trials') accuracy was relatively spared, indicating that the deployment of order of operations knowledge was selectively affected by the grouping pressures. Several aspects of the experiment make this influence of perception on algebra striking. First, they demonstrate a genuine cognitive illusion in the domain of mathematics. The criteria for cognitive illusions in reasoning are that people systematically show an influence of a factor in reasoning. The factor should normatively not be used, and people agree, when debriefed, that they were wrong to use the factor (Tversky and Kahneman 1974). The second impressive aspect of the results is that participants continued to show large influences of grouping on equation verification even though they received trial-by-trial feedback.

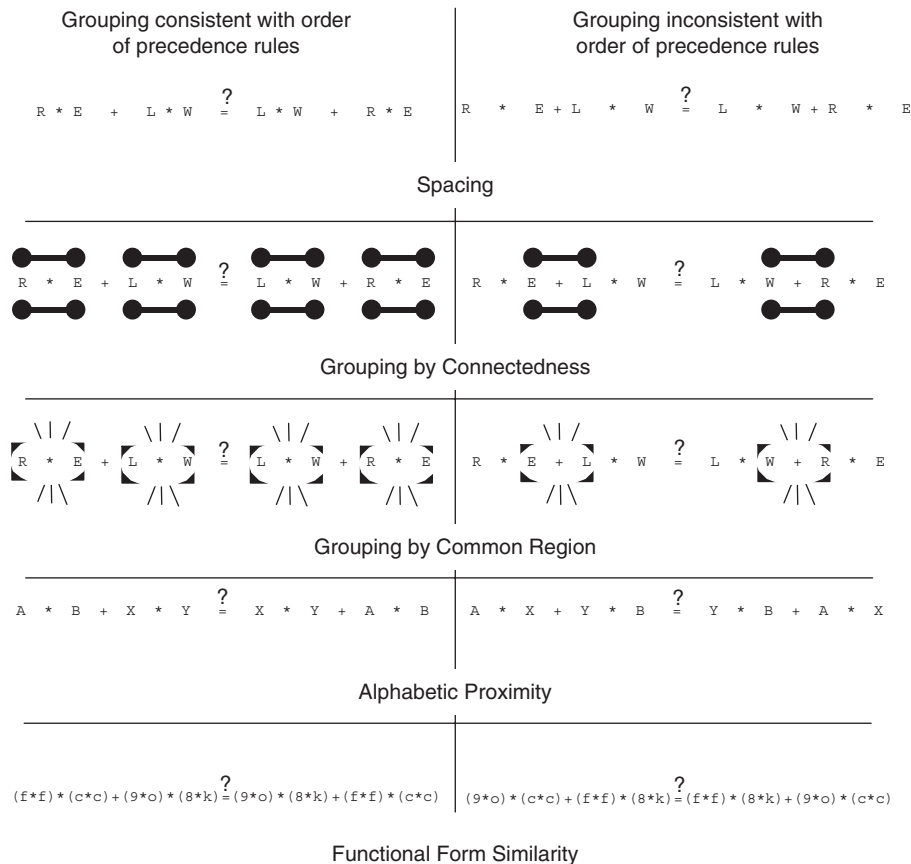


Fig. 16.7 Samples from five experiments reported by Landy and Goldstone (under review). Participants were asked to verify whether an equation is necessarily true. Grouping suggested by factors such as physical spacing, regions suggested by geometric forms, proximity in the alphabet, and functional form similarity could be either consistent or inconsistent with the order of precedence of arithmetical operators (e.g. multiplications are calculated before additions). The above equalities are all true, but participants make far more errors when the perceptual and form-based groupings are inconsistent rather than consistent.

Constant feedback did not eliminate the influence of the perceptual cues. This suggests that sensitivity to grouping is automatic or at least resistant to strategic, feedback-dependent control processes. The third impressive aspect of the results is that an influence of grouping is found in mathematical reasoning. Mathematical reasoning is often taken as a paradigmatic case of purely symbolic reasoning, more so even than language which, in its spoken form, is produced and comprehended before children even have formal operations (Inhelder and Piaget 1958). Algebra is, according to many people's intuition, the clearest case of widespread symbolic reasoning in all human cognition. Showing that perceptual factors influence even algebraic reasoning provides

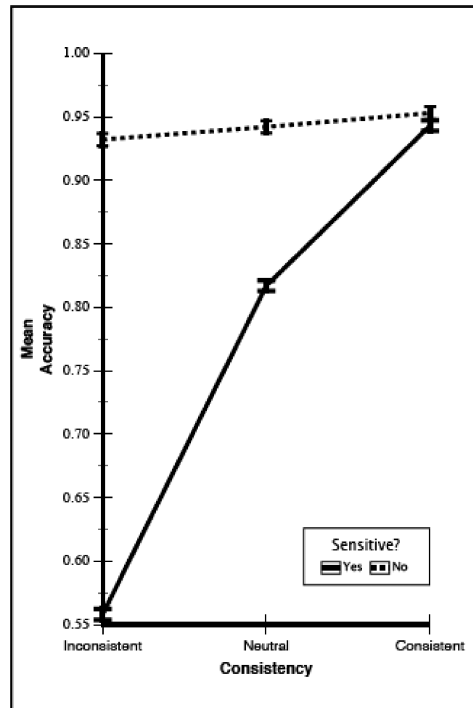


Fig. 16.8 Results from a manipulation of physical spacing in experiment 1 of Landy and Goldstone (under review). When spacing was inconsistent (right side of Figure 16.7) with the order of precedence, the error rate increased more than six-fold compared with when spacing was consistent. This effect was only found when equations were sensitive to physical spacing – trials where different consistent versus inconsistent answers would be given to the question, ‘Are the two sides of this equation necessarily equal?’ if a participant were influenced by spacing.

prima facie support for the premise that grounding cannot be ignored for *any* cognitive task.

Our further experiments have extended this observed influence of grouping factors on algebraic reasoning. The second and third rows of Figure 16.7 show manipulations of other Gestalt laws of perceptual organization. According to the principle of connectiveness (Palmer and Rock 1994), objects that are physically connected to one another have a tendency to be grouped together. Connecting the circles in Row 2 of Figure 16.7 causes them to be grouped together, and experimental results show that when they are grouped together, they tend to group the mathematical symbols with which they are vertically aligned. The third row shows that this grouping pressure does not require physical connection, but rather can also be formed through implied common region.

In both of these examples, even though participants know that the geometric forms are irrelevant to their mathematical task, when the geometric forms are consistent

with order of operations, far better algebraic performance results than if the forms are inconsistent. The fourth row of Figure 16.7 shows that alphabetic proximity has an analogous effect to physical proximity. Participants apparently have a tendency to group letters that are close in the alphabet. If close letters, like 'A' and 'B,' are related by an addition operator, this hinders algebraic performance; if the close letters are related by a multiplication operator, this facilitates performance. This influence of alphabetic proximity on mathematical reasoning is surprising because by standard formal accounts all variable names are equivalent and participants, in their explicit description of the task, endorse the statement that the letters used to designate variables are irrelevant. Finally, the last row shows an influence of the functional form of the terms being multiplied and added. The term ' $c * c$ ' has a form-based similarity to ' $f * f$ ' – they both involve a variable being multiplied by itself. This similarity is sufficient to bias participants to group these terms together. Again, this form-based group helps performance if the terms are related by multiplication, but hinders performance if they are related by addition.

The cumulative weight of these results indicates that even the highly symbolic activity of algebraic calculation is strongly affected by perceptual grouping. Most accounts of the importance of notation in mathematics, and indeed of symbols generally, hinge on their abstract character. Since symbols are largely considered amodal, symbolic reasoning is also easily assumed to be amodal; that is, symbolic reasoning is supposed to depend on internal structural rules which do not relate to explicit external forms. In contrast, the mathematical groupings that our participants create are heavily influenced by groupings based on perception and superficial similarities. The theoretical upshot of this work is to question the assumption that cognition operates generally like systems of algebra or mathematical logic. By traditional symbolic manipulation accounts, to cognize is to apply laws to structured strings where those laws generalize to the shape of the symbols, and that shape is arbitrarily related to the symbols' content (Fodor 1992).

The laws of algebra work in just the same manner. Multiplication is commutative no matter what terms are multiplied. Mathematical cognition could have worked like this too; if it had, it would have provided a convenient explanation of how people perform algebra (Anderson 2005). However, the conclusion from our experiments is that seeing ' $X + Y * A$ ' cannot trivially be translated into the amodal mentalesse code $*(+(G001, G002), G003)$. The physical spacing, superficial similarities between the letters, and background pictorial elements are all part of how people treat ' $X * A + Z$.' Although it would be awfully convenient if a computational model of algebraic reasoning could aptly assume the transduction from visual symbols to mental symbols, taking this for granted would leave all of our current experimental results a mystery. We must resist the temptation to posit mental representations with forms that match our intellectualized understanding of mathematics. A more apt input representation to give our future computational models would be a bitmap graphic of the screen that includes the visually presented symbols as well as their absolute positions, spacings, sizes, and accompanying nonmathematical pictorial elements, as well as the associations carried by particular shapes.

16.4 Conclusions

We have tried to develop an account of embodied cognition that is consistent with the goal of teaching scientific and mathematical knowledge that is transportable. Toward this end, our chain of argumentation includes several links:

- ◆ We argued that understanding complex systems principles is important, both scientifically and pedagogically. Seemingly unrelated systems are often deeply isomorphic, and the mind that is prepared to use this isomorphism can borrow from understanding a concrete scenario.
- ◆ If one is committed to fostering the productive understanding of complex systems, then one must be interested in promoting knowledge that can be transported across disciplinary boundaries.
- ◆ Our observation of students interacting with complex systems simulations indicates that one of the most powerful educational strategies is to have students actively interpret perceptually present situations. A situation's events inform and correct interpretations, while the interpretations give meaning to the events.
- ◆ Perspective-dependent interpretations can promote transfer where formalism-centered strategies fail, by educating people's flexible perception of similarity. Transfer, by this approach, occurs not by applying a rule from one domain to a new domain, but rather by allowing two scenarios to be seen as embodying the same principle.
- ◆ Even algebraic reasoning is sensitive to perceptual grouping factors, suggesting that perhaps all cognition may intrinsically involve perceptually grounded processes.

Considerable psychological evidence indicates that far transfer of learned principles is often difficult and may only reliably occur when people are explicitly reminded of the relevance of their early experience when confronted with a subsequent related situation (Gick and Holyoak 1980, 1983). This body of evidence stands in stark contrast to other evidence suggesting that people automatically and unconsciously interpret their world in a manner that is consistent with their earlier experiences (Kolers and Roediger 1984; Roediger and Geraci 2005). These statements are, however, reconcilable. A lasting influence of early experiences on later experiences occurs when perceptual systems have been transformed. A later experience will be inevitably processed by the systems that have been transformed by an initial experience, and priming naturally occurs. When a principle is simply grafted onto an early experience but does not change how the experience is processed, there is little chance that the principle will come to mind when it arises again in a new guise. The moral is clear: to reliably make an interpretation come to mind, one needs to affect how a situation is interpreted as it is 'fed forward' through the perceptual system rather than tacking on the interpretation at the end of processing.

An embodied cognition perspective offers promise of scientifically-grounded educational reform in both of our domains of empirical consideration: complex systems simulations and algebraic calculation. In particular, reforms are advocated that involve changes to perception, or the co-option of natural perceptual processes for tasks

requiring symbolic reasoning. From this perspective, it is striking that developing expertise in most scientific domains involves perceptual learning. Biology students learn to identify cell structures, geology students learn to identify rock samples, and chemistry students learn to recognize chemical compounds by their molecular structures. In mathematics, perceptually familiar patterns are a major source of solutions for solving novel problems. Progress in the teaching of these fields depends on understanding the mechanisms by which perceptual and conceptual representations mutually inform and interpenetrate one another. Specific hypotheses for future exploration stem from an embodied perspective. How can we modify the perceptual aspects of mathematical notation conventions to cue students as to their formal properties? The minus sign ‘-’ is just as symmetrical as the plus sign ‘+’, but subtraction is order-dependent in a way that addition is not. Should the signs reflect this formal difference? Can physical spacing be used to scaffold learning formal mathematical properties? Can we help students learn the order of precedence by presenting them with ‘A + X*Y’ before giving them the more neutral notation of ‘A + B * C?’

More generally, our analyses of science and mathematics have led us to reconsider what ‘abstract cognition’ entails. Abstract cognition should be interpreted akin to abstract art. Abstract art is still based in perception – it would not be art if it couldn’t be seen. Similarly, for cognition, abstract is not the opposite of perceptual, nor are ‘perceptual’ and ‘superficial’ synonyms. A fertile and novel perspective on abstraction may be that it is the process of a perceptual system interpreting a situation in a useful manner. Interpretation presupposes that there is an external situation to interpret. Oftentimes, the perceptual system will need to be trained in order to avoid superficial or misleading attributes. We believe that viewing abstraction as the deployment of trained and strategic perceptual/motor processes can go a long way toward demystifying symbolic thought, hopefully leading to the development of robust and working models of cognition.

Debate

Arthur Graesser: Part of the problem with transfer from one science domain to another is that the problems may be rather different. If you use one solution you may have an isomorphic problem, but you are really imposing it, trying to make sense of it. If it is good enough, you may say that there is an analogy, and it is that process that helps you perceive things. Given the trajectory of how people make analogies, how would you interpret your data?

Robert Goldstone: You do not want to have rampant analogy making if there are real, important differences between domains. In part, what I am arguing for is not just the cognitive science of transfer, but I am also making claims about what science pedagogy should look like. What are the principles that should be taught in our schools? There I would make the claim for teaching the kinds of complex systems principles that I have described. So, I do agree with you that it is problematic when you have principles that apply in one domain but not another and you try to shoe-horn them into alignment. That clearly happens. But at the same time,

exactly the same formalisms can often be successfully applied in different domains. For example, I think some of the most important lessons that we should be teaching in our science classes concern positive and negative feedback systems. Naturally, the nature of these systems will need to be tailored to a scenario, but these skeletal structures arise time after time. Teaching these principles can be a lot more valuable than teaching a student the name for some amino acid because these are principles that at least have the possibility of being relevant for a new situation that a student is likely to come across at a later point. I believe there are a number of such principles: oscillations, reaction–diffusion systems where the presence of one element gives rise to more of a second element that may destroy the first element at the same time as the first element is spreading out, or lateral inhibition between neighbouring elements at the same layer in a network. These principles are general and do arise in different guises. It would be wrong to leave things at that level of description and say that we have completely explained the system. We then need to adapt the characterization to the specifics of the scenario, just as case-based reasoning researchers would have us do.

Max Louwerse: I want to challenge your conclusions that just because something is formal reasoning doesn't make it amodal. I think that's wrong. I think that if it is formal reasoning then it is amodal. What you've pointed to is that there could be performance problems. So, I would want to change the claim to – just because something is formal reasoning does not mean that perception can't affect performance. If I'm teaching predicate calculus in a logic class I'll tell students, 'Here's a rule, *modus ponens*, that is sensitive only to the shapes of symbols.' But if I make the material implication symbol a mile long so that students have to walk several blocks to see what the consequent of the material implication is, they are not going to do as well because they have already forgotten the antecedent. But they engage in formal reasoning when they do reason.

Goldstone: I'm really skeptical of the performance/competence distinction. When you are doing formal reasoning, it is just physical manipulation of chicken scrawls. When you learn logic you're learning that you cross this bit out and replace it with another kind of chicken scrawl. I think, very literally, it is still just concrete and perceptual. You're just learning different types of transformation and translation procedures. In our experiments, we don't just get response-time differences, we get accuracy differences. If you're thinking about it from a formal symbol systems perspective, you would need to say that people are coming up with different algebraic tree structures based upon perceptual properties.

Louwerse: Why is it wrong to think that what I'm teaching my students to do is to behave like a Turing machine? Do you think that a Turing machine would show the same kinds of performance problems that a human being would?

Goldstone: I think it's revealing that Alan Turing described the Turing machine in a particular physical context. He wasn't describing it only in terms of a pure mathematical formalism. He was talking about a specific kind of machine that had specific

properties, such as tape heads that are moving back and forth. So, I will take the more radical position that what we are talking about as formal symbol manipulation is what Newell and Simon referred to as physical symbol systems. They are physical. They are based upon the form of the objects. So it matters what those forms are and how they are processed. This relates to something that Friedemann [Pulvermüller] discussed (see Chapter 6). Word representations activate Broca's area because part of their representation is their production. It's not as though words are formal systems. They are being produced and understood as scrawls or sound waves.

Friedemann Pulvermüller: Thanks very much for this important contribution. I want to add a little thing. I'm fully with you in everything you say, especially with regard to education. I'm coming from a neuropsychological domain called aphasia therapy research, and teaching patients language after lesions. It is disastrous to see that the majority of therapists would present words and pictures in a far-removed situation in a 'naming game'. A neurophysiologist had a patient with global aphasia, and in testing her he asked, 'Who is this person sitting to your left?' (it was the patient's daughter). She responded '.. I can't come up with it. I'm sorry, I have tried and tried ... My poor Jacqueline, I can't even remember your name.' This is exactly the point you are making. The situation and community of intentions count. The language game that is being played is very important. So it is not just the relation between the picture and word that counts. Noncommunicative language wouldn't even produce the same speech output. With regard to the competence/performance distinction, if we think of the brain in terms of real neuronal networks, we may have difficulty preserving the distinction. It is clear that the neural assemblies and their connections are the implementation of both the rules and the words – the competence of the system. The activity of these same networks is the correlate of performance. So, this distinction does not make too much sense.

Goldstone: The only thing that I would add to that is that, although I am skeptical of a performance/competence distinction, I'm all in favour of having responses based on different types of representations, giving rise to different performances on different occasions. Some of these representations will give rise to output that look more in accord with what we expect by our intellectualized understanding of logic. But I think it would be wrong to say that those that do correspond to our mathematical understanding are not done by cognitive, physical manipulations.

Deb Roy: I'd like to stay on the topic of performance versus competence and the question of whether we can keep this distinction or not. Of course, probably everyone here is thinking of Chomsky when we think about this distinction. David Marr used slightly different terminology, but his general idea was that when one is trying to understand a system, usually a complex system, it is useful to distinguish what it is doing from how it is doing it. If you don't have a clear idea of what it is doing then it is problematic to ask how it is doing it. So, if we get rid of the labels 'performance' and 'competence' and rephrase the question to ask, 'Is it still worthwhile to keep Marr's distinction?' Or, would you eliminate that distinction as well?

Goldstone: I'm happier preserving a 'what' versus 'how' distinction, and I also still want to be able to talk about errors in processing. We want to be able to say things like, 'What we want to get out is a three-dimensional representation of a scene, but if we use this particular stereopsis algorithm that combines information from the two eyes, then we won't get the desired output.' That is, we do need to be able to say that the operation of an algorithm falls short of its intended computation.

Roy: I'd say that is completely in line with the original performance/competence distinction. You're still on board, but just using different terminology.

Luc Steels: There was more to the original distinction. In a usage-based approach to language, the actual variation and performance is influenced all the time.

Roy: So you're saying that that performance versus competence assumes dynamic versus static systems?

Steels: You can tease out different meanings, but 'competence' ended up being a description of an abstract system without worrying about how using the system constantly impacts the original system.

Roy: Maybe there is a family of interpretations. When I think of Marr's 'what' level, I never imagined that it needs to be fixed. When one thinks of Chomsky, one may think that competence is innate and does not change perhaps.

Goldstone: For me, what is added with the performance/competence distinction is that it suggests that it is useful to ask, 'What would this system be able to do if you took away all of the contextualizing variables that Friedemann just described, or all of the system's performance-based limitations?' However, from my perspective, it's exactly because of those contextual and performance-based factors that the system is able to do anything at all. It doesn't make sense to talk about what the competence of the system would be after extracting out contextual and physical considerations. However, it still might make sense to ask, from a teleological perspective, what the aim of a system is – Marr's 'what is it doing?' question.

Roy: I can live with that.

Author note

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