

Convergence Analysis of the Stop-and-Go Blind Equalization Algorithm

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Abstract— Unlike traditional trained channel equalizers, not much work has been done to theoretically characterize the convergence properties of blind channel equalizers due to their inherent nonlinearity. It is only recently that convergence properties of some well-known algorithms such as GSA and CMA have been analytically derived. In this paper, the convergence properties of the stop-and-go algorithm proposed by Picchi and Prati are analyzed. The derived mean squared error and the coefficient trajectories are compared with simulation results to verify the validity of the analytical results.

Index Terms— Equalizers, iterative methods.

I. INTRODUCTION

CHANNEL equalization without the aid of a training sequence is referred to as blind channel equalization. Unlike traditional trained equalization algorithms, many of the widely employed blind equalization algorithms employ a nonlinearity at the equalizer output to generate the error signal for coefficient updates. Due to this nonlinearity, the analysis of the convergence properties of the blind equalization algorithms is usually difficult and it is only recently that analytical derivations of the convergence properties of some of the well-known blind equalization algorithms such as the Generalized Sato Algorithm (GSA) and the Constant Modulus Algorithm (CMA) were published in the literature [4], [5].

A major drawback of the GSA and the CMA is that the residual error after convergence is unacceptably large [1]. An improved blind equalization technique for handling the large residual error problem is the stop-and-go algorithm proposed by Picchi and Prati [2]. The convergence properties of the stop-and-go algorithm will be derived in this paper.

For the baud-rate feedforward linear equalizer adopting the stop-and-go algorithm, the filter coefficient update equation is given as $\underline{c}(n+1) = \underline{c}(n) + \alpha \underline{x}(n) \cdot e^{\text{SG}}(n)^*$ where α is the adaptation step size, $\underline{c}^T(n) = [c_0(n), c_1(n), \dots, c_{N-1}(n)]$ is the coefficient vector, $\underline{x}^T(n) = [x(n), x(n-1), \dots, x(n-N+1)]$ is the input vector, $e^{\text{SG}}(n)$, which is produced by processing the equalizer output $z(n)$, is the error signal at time n , and N is the number of the equalizer tap coefficients.

The stop-and-go algorithm uses the decision directed (DD) error signal and allows coefficient updates only when the DD error signal is considered to be reliable. The DD error signal is given by $\hat{e}(n) = \hat{a}(n-D) - z(n)$ where $\hat{a}(n-D)$ is

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TABLE I
 $s(x)$ FOR THE SQUARE 16-QAM DATA CONSTELLATION
 WITH DATA LEVELS AT ± 1 AND ± 3

condition		$s(x)$
$x \leq -2$	and $\text{sgn}(-\beta^{\text{SG}} - x) = \text{sgn}(-3 - x)$	$-3 - x$
$-2 < x \leq 0$	and $\text{sgn}(-\beta^{\text{SG}} - x) = \text{sgn}(-1 - x)$	$-1 - x$
$0 < x \leq 2$	and $\text{sgn}(\beta^{\text{SG}} - x) = \text{sgn}(1 - x)$	$+1 - x$
$x < 2$	and $\text{sgn}(\beta^{\text{SG}} - x) = \text{sgn}(3 - x)$	$+3 - x$
otherwise		0

the decision made on the data symbol $a(n-D)$ from $z(n)$, and D is the overall propagation delay of the channel and the equalizer. A method of judging the reliability of the DD error signal proposed in [2] is to see whether the sign of the DD error coincides with a Sato-like error $\tilde{e}(n) = \beta^{\text{SG}} c \text{sgn}(z(n)) - z(n)$ where $c \text{sgn}(x) = \text{sgn}(x_R) + j \text{sgn}(x_I)$ and β^{SG} is a suitable real value chosen depending on the data constellation. Hence, the real and imaginary parts of the error signal for the stop-and-go algorithm are given as follows:

$$e_\gamma^{\text{SG}}(n) = f_\gamma(n) \hat{e}_\gamma(n), \quad \text{for } \gamma = I, R \quad (1)$$

where

$$f_\gamma(n) = \begin{cases} 1, & \text{if } \text{sgn}(\tilde{e}_\gamma(n)) = \text{sgn}(\hat{e}_\gamma(n)) \\ 0, & \text{otherwise for } \gamma = I, R \end{cases} \quad (2)$$

are the flags that determine whether or not coefficient update should be performed.

Considering both the intersymbol interference and the channel noise, the equalizer input signal may be written as $x(n) = \sum_{i=0}^{N-1} a(n-i)h_i + N(n)$ where $a(n)$ is the transmitted data, $h_i, i = 0, 1, \dots, N-1$, is the impulse response of the channel, and $N(n)$ is white Gaussian noise with $E\{|N(n)|^2\} = P_N$. Though the following computation examples and numerical results are given for a square 16-QAM constellation with data levels at ± 1 and ± 3 , results in this paper may be applied to the convergence analysis of other QAM constellations. For the 16-QAM data constellation, we may write $e^{\text{SG}}(n) \equiv S(z(n)) = s(z_R(n)) + js(z_I(n))$ where $s(\cdot)$ is given by Table I.

A common method to ease the analysis of convergence properties of equalizers is to orthogonalize the equalizer input using a similarity transformation [3]–[5]. The input covariance matrix $R = E\{\underline{x}(n)\underline{x}^H(n)\}$ is Hermitian symmetric and can be decomposed as $R = U^H \Lambda U$ where Λ is a diagonal matrix whose diagonal elements are the eigenvalues of R . Then, the transformed tap-coefficient update equation is given as $\underline{w}(n+1) = \underline{w}(n) + \alpha \underline{y}(n) S^*(z(n))$ where $\underline{w}(n) = U \underline{c}(n)$, $\underline{y}(n) = U \underline{x}(n)$, and $S^*(z(n)) = s(z_R(n)) - js(z_I(n))$. For the analysis, we employ the assumptions 1)–4) in [4] and we refer the readers to [4] for a detailed discussion of the assumptions. Note that the assumption 5) in [4], which

approximates $\text{var}\{z(n) | d(n)\}$ with MSE,¹ is not used in this paper.

II. CONVERGENCE ANALYSIS

In this section, we will derive the equations for the mean squared error (MSE) and the tap-coefficient trajectories for the system under consideration. Using assumptions 1) and 4) of [4], the output MSE $E\{|\varepsilon(n)|^2\}$ may be approximated as a function of the first- and the second-order moments of the equalizer tap coefficients as

$$E\{|\varepsilon(n)|^2\} = E\{|d(n) - z(n)|^2\} \\ \approx P_a(1 - 2\text{Re}[\underline{M}^H(n)\underline{\eta}]) + \sum_{k=0}^{N-1} \Gamma_k(n)\Lambda_{kk} \quad (3)$$

where $d(n) = a(n - D)$, $P_a = E\{|a(n)|^2\}$, $M_k(n) = E\{w_k(n)\}$, $\Gamma_k(n) = E\{|w_k(n)|^2\}$, $k = 0, \dots, N - 1$, $\underline{\eta} = U\underline{b}$, and $\underline{b} = [h_D, h_{D-1}, \dots, h_{D-N+1}]^T$. The recursive equations for $M_k(n)$ and $\Gamma_k(n)$ needed to compute the MSE trajectory using (3) are given as

$$M_k(n+1) = M_k(n) + \alpha E\{y_k(n)S^*(z(n))\} \quad (4)$$

$$\Gamma_k(n+1) = \Gamma_k(n) + 2\alpha E\{\text{Re}[w_k^*(n)y_k(n)S^*(z(n))]\} \\ + \alpha^2 E\{|y_k(n)|^2|S^*(z(n))|^2\}, \\ \text{for } k = 0, 1, \dots, N - 1. \quad (5)$$

Computation of the expectations in (4) and (5) is based on the assumption that $z(n)$ and $\underline{y}(n)$ conditioned on $d(n)$ and $\underline{w}(n)$ are jointly Gaussian [4].

Defining $p(n) \equiv \underline{M}^H(n)\underline{\eta}$ and using approximation (3.16) and Assumption 4) of [4], we get $E\{z(n) | d(n), \underline{w}(n)\} \approx d(n)p(n)$, $E\{\underline{y}(n) | d(n), \underline{w}(n)\} = d(n)\underline{\eta}$, $\text{cov}\{\underline{y}(n), \underline{y}(n) | d(n), \underline{w}(n)\} = \Lambda - P_a\underline{\eta}\underline{\eta}^H \equiv C$ and $\text{cov}\{\underline{y}(n), z(n) | d(n), \underline{w}(n)\} = C\underline{w}(n) \equiv \underline{\Phi}(n)$. Using this and the results in Appendixes A and B, we may compute the conditional expectations $E\{y_k(n)S^*(z(n)) | d(n), \underline{w}(n)\}$, $E\{w_k^*(n)y_k^*(n)S^*(z(n)) | d(n), \underline{w}(n)\}$, and $E\{|y_k(n)|^2|S^*(z(n))|^2 | d(n), \underline{w}(n)\}$. Averaging these conditional expectations over the conditioning variables gives

$$E\{y_k(n)S^*(z(n))\} = \eta_k E\{d(n)g_0^*\} + \frac{[CM(n)]_k}{\sigma_z^2(n)} B_0 \quad (6)$$

$$E\{w_k^*(n)y_k(n)S^*(z(n))\} = M_k^*(n)\eta_k E\{d(n)g_0^*\} \\ + \frac{E\{\Phi_k(n)w_k^*(n)\}}{\sigma_z^2(n)} B_0 \quad (7)$$

$$E\{|y_k(n)|^2|S^*(z(n))|^2\} = B_1 + 2\text{Re}\left[\frac{[CM(n)]_k\eta_k^*}{\sigma_z^2(n)} B_2\right] \\ + \frac{E\{|\Phi_k(n)|^2\}}{\sigma_z^4(n)} B_3 \quad (8)$$

where B_i , $i = 0, 1, 2, 3$, are given by $B_0 = E\{g_1\} - p(n)E\{d(n)g_0^*\}$, $B_1 = C_{kk}E\{g_2\} + |\eta_k|^2 \cdot E\{|d(n)|^2g_2\}$, $B_2 = E\{d^*(n)g_3\} - p(n)E\{|d(n)|^2g_2\}$, $B_3 = E\{g_4\} + |p(n)|^2E\{|d(n)|^2g_2\} - \sigma_z^2(n)E\{g_2\} - 2\text{Re}[p^*(n)E\{d^*(n)g_3\}]$.

¹This approximation introduces an error when the channel significantly rotates the constellation [5].

$g_3\}$, and g_i , $i = 0, 1, \dots, 4$, are given by $g_0 = E\{S(Z)\}$, $g_1 = E\{ZS^*(Z)\}$, $g_2 = E\{|S(Z)|^2\}$, $g_3 = E\{Z|S(Z)|^2\}$, $g_4 = E\{|Z|^2|S(Z)|^2\}$ where Z is a complex Gaussian random variable with mean $d(n)p(n)$ and variance $\sigma_z^2(n)$. An example of the evaluation of the g_i 's for the case when $\beta^{\text{SG}} = 3$ is given in Appendix C. The expectations in (7) and (8) are given by $E\{\Phi_k(n)w_k^*(n)\} = C_{kk}\Gamma_k(n) + M_k^*(n)\sum_{i \neq k} C_{ki}M_i(n)$ and $E\{|\Phi_k(n)|^2\} = \sum_{i=0}^{N-1} |C_{ki}|^2(\Gamma_i(n) - |M_i(n)|^2) + |\sum_{i=0}^{N-1} C_{ki}M_i(n)|^2$. The recursive equations for $M_k(n)$ and $\Gamma_k(n)$ given by (4) and (5) together with the MSE expression in (3) allow us to compute the transient behavior of the MSE and the tap coefficients for the stop-and-go algorithm.

III. NUMERICAL RESULTS

Here, we present the computer simulation and the numerical results for the square 16-QAM data constellation with data levels at ± 1 and ± 3 and β^{SG} of three. A 21-tap baud-rate equalizer was used and initialized with the center tap coefficient ($c_{10}(0)$) set to unity and the rest to zero. According to this initial tap assignment, the first and the second moments of the transformed tap coefficients were initialized as $\underline{M}(0) = U\underline{c}(0)$ and $\Gamma_k(0) = |M_k(0)|^2$, $k = 0, \dots, N - 1$. The overall delay of the channel and the equalizer was set to $D = 20$. The trajectories of $M_k(n)$ and $\Gamma_k(n)$ were derived using (4) and (5), respectively, and the MSE trajectory was calculated using (3).

Fig. 1(a) shows the simulated and analytically computed MSE trajectories for channel 1, which is the same as the channel 1 in [4], with $\alpha = 0.0005$. The simulation results were obtained by ensemble averaging over 100 independent simulation runs. We observe that the MSE trajectories predicted by the analysis very well matches the simulation results. Fig. 1(b) shows the mean trajectories of the real part of the center tap coefficient. Both the simulation and the analytic results converge to 0.605, which corresponds to the optimum Wiener tap coefficient.

We have carried out a similar set of simulations for a case when the channel significantly rotates the constellation. Fig. 2 shows the results for a second channel model with \underline{h} given in Table II. We observe that the accuracy of the analysis does not degrade when the channel rotates constellation.

IV. CONCLUSION

Analytic expressions for predicting the MSE and the mean tap coefficient trajectories of the stop-and-go blind equalization algorithm were derived and their accuracy was evaluated via computer simulations. The analytic results may also be used to test the convergence of the stop-and-go algorithm and optimize the equalizer parameters such as the step size α for a given channel.

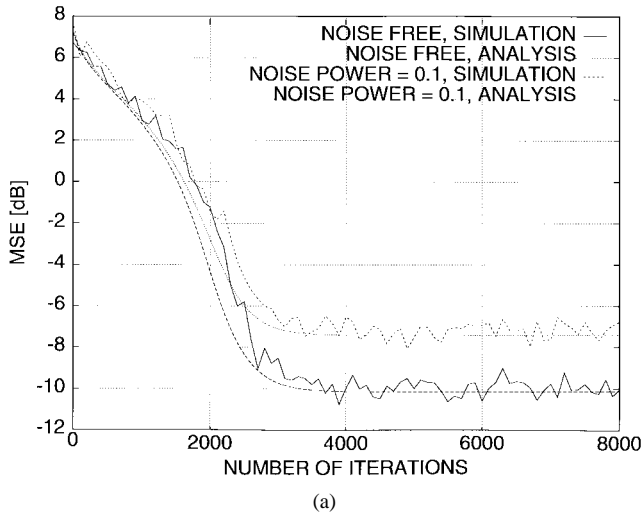
APPENDIX A

EVALUATION OF $\text{var}\{z(n) | d(n), \underline{w}(n)\}$

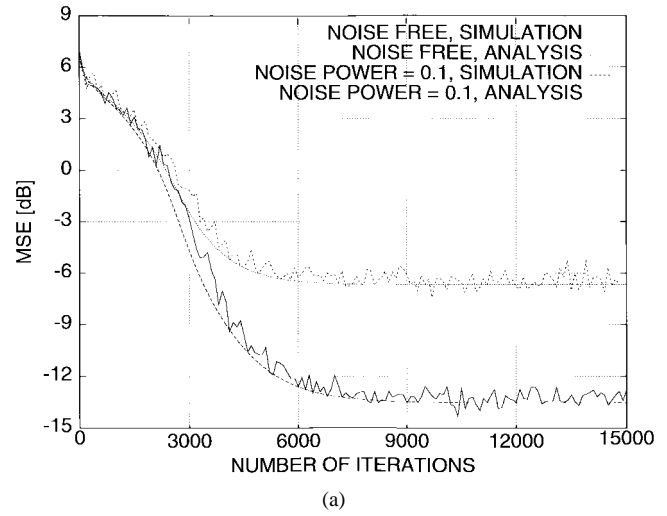
Using Assumption 1) in [4], we get

$$E\{\underline{y}(n) | d(n)\} = d(n)\underline{\eta} \quad (A.1)$$

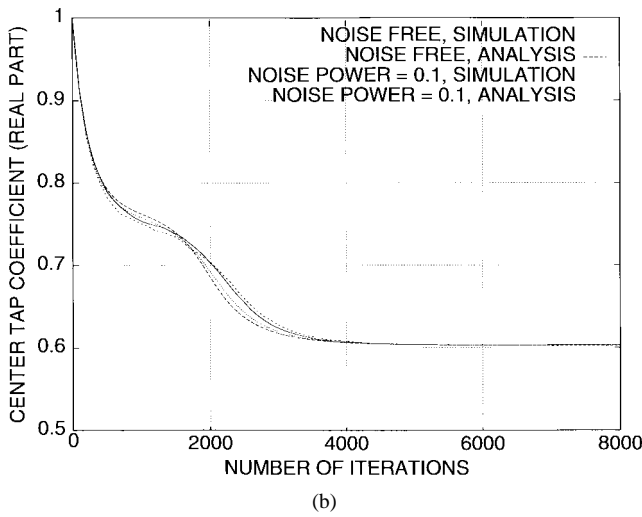
$$E\{\underline{y}(n)\underline{y}^H(n) | d(n)\} = \Lambda - (|d(n)|^2 - P_a)\underline{\eta}\underline{\eta}^H. \quad (A.2)$$



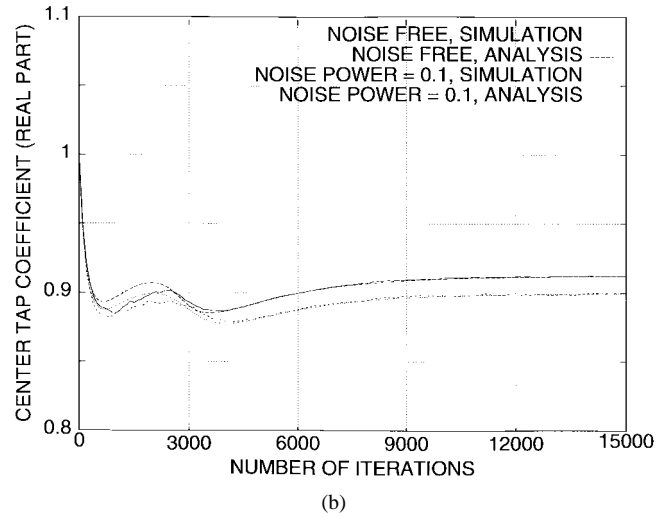
(a)



(a)



(b)



(b)

 Fig. 1. Trajectories for channel 1, $\alpha = 0.0005$, $P_N = 0$ and 0.1. (a) MSE and (b) center tap-coefficient (real part).

 Fig. 2. Trajectories for channel 2, $\alpha = 0.0005$, $P_N = 0$ and 0.1. (a) MSE and (b) center tap-coefficient (real part).

Using (A.1) and (A.2) and that $z(n) = \underline{w}^H(n)\underline{\eta}$, we can compute

$$\begin{aligned} \text{var}\{z(n) \mid d(n), \underline{w}(n)\} \\ = \sum_{i=0}^{N-1} \lambda_i |w_i(n)|^2 - P_a \left| \sum_{i=0}^{N-1} w_i^*(n) \eta_i \right|^2. \end{aligned} \quad (\text{A.3})$$

When the step size α is small so that the variations of $\underline{w}(n)$ about its mean value may be ignored, we can assume that the trajectory of $\underline{w}(n)$ is approximately deterministic and is given by mean value trajectory of $\underline{w}(n)$ [4]. Thus the equalizer output conditioned on $a(n)$ is also a complex Gaussian random variable with

$$\begin{aligned} \text{var}\{z(n) \mid a(n)\} &\equiv \sigma_z^2(n) \\ &= \sum_{i=0}^{N-1} \lambda_i \Gamma_i(n) - P_a \left| \underline{M}^H(n) \underline{\eta} \right|^2 \\ &\quad + P_a \sum_{i=0}^{N-1} (\Gamma_i(n) - |M_i(n)|^2) |\eta_k|^2. \end{aligned} \quad (\text{A.4})$$

 TABLE II
 IMPULSE RESPONSE OF CHANNEL 2

k	0 ... 7	8	9	10	11	12	13	14 ... 20
$h_{k,r}$	0	0.04	-0.124	0.854	-0.208	0.249	-0.16	0
$h_{k,l}$	0	0.03	-0.104	0.420	0.273	-0.174	0.02	0

APPENDIX B

 EVALUATION OF $E\{Y S^*(Z)\}$ AND $E\{|Y|^2 |S(Z)|^2\}$

Let Y, Z be jointly Gaussian complex random variables with means, variances, and covariance given by $E\{Y\} = \mu_Y$, $E\{Z\} = \mu_Z$, $\text{var}\{Y\} = \sigma_Y^2$, $\text{var}\{Z\} = \sigma_Z^2$, and $\text{cov}\{Y, Z\} = C$. Note that we may write

$$E\{Y S^*(Z)\} = E\{Y s(Z_R)\} - j E\{Y s(Z_I)\} \quad (\text{A.5})$$

$$E\{|Y|^2 |S(Z)|^2\} = E\{|Y|^2 [s(Z_R)]^2\} + E\{|Y|^2 [s(Z_I)]^2\} \quad (\text{A.6})$$

which implies that $E\{Y S^*(Z)\}$ and $E\{|Y|^2 |S(Z)|^2\}$ do not depend on the correlation coefficient between Z_R and Z_I . Hence, we may assume that Z_R and Z_I are uncorrelated, which simplifies the following presentation.

Using the above assumption, the conditional mean and variance of Y given Z are given as $E\{Y | Z\} = \mu_Y + (C/\sigma_Z^2)(Z - \mu_Z)$ and $\text{var}\{Y | Z\} = \sigma_Y^2 - (|C|^2/\sigma_Z^2)$ from which we may compute

$$\begin{aligned} E\{YS^*(Z)\} &= E\{S^*(Z)E\{Y | Z\}\} \\ &= \mu_Y E\{S^*(Z)\} + \frac{C}{\sigma_Z^2} E\{ZS^*(Z)\} - \frac{C}{\sigma_Z^2} \mu_Z E\{S^*(Z)\} \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} E\{|Y|^2 | S(Z)|^2\} &= E\left\{|S(Z)|^2 \left(\text{var}\{Y | Z\} + |E\{Y | Z\}|^2\right)\right\} \\ &= (\sigma_Y^2 + |\mu_Y|^2) E\{|S(Z)|^2\} \\ &\quad + 2\text{Re}\left[\frac{C}{\sigma_Z^2} \mu_Y^* (E\{Z|S(Z)|^2\} - \mu_Z E\{|S(Z)|^2\})\right] \\ &\quad + \frac{|C|^2}{\sigma_Z^4} (E\{|Z|^2 | S(Z)|^2\} - 2\text{Re}[\mu_Z^* E\{Z|S(Z)|^2\}]) \\ &\quad + \frac{|C|^2}{\sigma_Z^4} (|\mu_Z|^2 - \sigma_Z^2) E\{|S(Z)|^2\}. \end{aligned} \quad (\text{A.8})$$

APPENDIX C

EVALUATION OF THE g_i 's FOR $\beta^{\text{SG}} = 3$

Let $Z = Z_R + jZ_I$ be a complex Gaussian random variable with mean and variance given by $E\{Z\} = \mu_R + j\mu_I$ and $\text{var}\{Z_R\} = \text{var}\{Z_I\} = \frac{1}{2}\text{var}\{Z\} = \sigma^2$. Let us define $G_\gamma^{ij}(\cdot)$, H_γ^{ij} and $F_n(\cdot)$ as follows:

$$G_\gamma^{ij}(a, b, c) \equiv \int_a^b x^i (c-x)^j \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_\gamma}{\sigma}\right)^2\right] dx, \quad \text{for } \gamma = R, I, \quad j = 1, 2 \quad \text{and} \quad 0 \leq i < j \quad (\text{A.9})$$

$$\begin{aligned} H_\gamma^{ij} &\equiv G_\gamma^{ij}(-\infty, -2, -3) + G_\gamma^{ij}(-1, 0, -1) \\ &\quad + G_\gamma^{ij}(0, 1, 1) + G_\gamma^{ij}(2, +\infty, 3) \end{aligned} \quad (\text{A.10})$$

$$F_n(x, y) \equiv \int_x^y \frac{t^n}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt, \quad \text{for } n = 0, 1, 2, 3, 4. \quad (\text{A.11})$$

Then, the g_i 's can be written in terms of H_γ^{ij} 's as $g_0 = H_R^{01} + jH_I^{01}$, $g_1 = H_R^{11} + H_I^{11} + j(\mu_I H_R^{01} - \mu_R H_I^{01})$, $g_2 = H_R^{02} + H_I^{02}$, $g_3 = H_R^{12} + \mu_R H_I^{02} + j(H_I^{12} + \mu_I H_R^{02})$, $g_4 = H_R^{22} + H_I^{22} + (\sigma_R^2 + \mu_R^2)H_I^{02} + (\sigma_I^2 + \mu_I^2)H_R^{02}$.

The $G_\gamma^{ij}(\cdot)$'s, which are needed for the calculation of the H_γ^{ij} 's, may be expressed in terms of $F_n(\cdot)$'s as

$$G_\gamma^{01}(a, b, c) = -\sigma F_1(a_\gamma, b_\gamma) + (c - \mu_\gamma) F_0(a_\gamma, b_\gamma) \quad (\text{A.12})$$

$$\begin{aligned} G_\gamma^{11}(a, b, c) &= -\sigma^2 F_2(a_\gamma, b_\gamma) + (c - 2\mu_\gamma)\sigma F_1(a_\gamma, b_\gamma) \\ &\quad + (c - \mu_\gamma)\mu_\gamma F_0(a_\gamma, b_\gamma) \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} G_\gamma^{02}(a, b, c) &= \sigma^2 F_2(a_\gamma, b_\gamma) + 2\sigma(\mu_\gamma - c)F_1(a_\gamma, b_\gamma) \\ &\quad + (\mu_\gamma - c)^2 F_0(a_\gamma, b_\gamma) \end{aligned} \quad (\text{A.14})$$

$$\begin{aligned} G_\gamma^{12}(a, b, c) &= \sigma^3 F_3(a_\gamma, b_\gamma) + \sigma^2(3\mu_\gamma - 2c)F_2(a_\gamma, b_\gamma) \\ &\quad + \sigma(\mu_\gamma - c)(3\mu_\gamma - c)F_1(a_\gamma, b_\gamma) \\ &\quad + \mu_\gamma(\mu_\gamma - c)^2 F_0(a_\gamma, b_\gamma) \end{aligned} \quad (\text{A.15})$$

$$\begin{aligned} G_\gamma^{22}(a, b, c) &= \sigma^4 F_4(a_\gamma, b_\gamma) + 2\sigma^3(2\mu_\gamma - c)F_3(a_\gamma, b_\gamma) \\ &\quad + \sigma^2(6\mu_\gamma^2 - 6\mu_\gamma c + c^2)F_2(a_\gamma, b_\gamma) \\ &\quad + 2\sigma\mu_\gamma(\mu_\gamma - c)(2\mu_\gamma - c)F_1(a_\gamma, b_\gamma) \\ &\quad + \mu_\gamma^2(\mu_\gamma - c)^2 F_0(a_\gamma, b_\gamma) \end{aligned} \quad (\text{A.16})$$

where $a_\gamma = (a - \mu_\gamma)/\sigma$ and $b_\gamma = (b - \mu_\gamma)/\sigma$. Finally, the $F_n(\cdot)$'s are given by $F_0(x, y) = Q(x) - Q(y)$, $F_1(x, y) = \frac{1}{\sqrt{2\pi}}(e^{-\frac{x^2}{2}} - e^{-\frac{y^2}{2}})$ and, for $n \geq 2$,

$$\begin{aligned} F_n(x, y) &= \frac{1}{\sqrt{2\pi}}(x^{n-1}e^{-\frac{x^2}{2}} - y^{n-1}e^{-\frac{y^2}{2}}) \\ &\quad + (n-1)F_{n-2}(x, y) \end{aligned} \quad (\text{A.17})$$

where $Q(\cdot)$ is the standard Q -function defined as $Q(x) \equiv \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$.

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