Introductory Lecture Notes on Nuclear and Particle Physics

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Abstract

These lecture notes are meant to introduce the new students to nuclear and particle physics research. Students will typically have had some exposure to physics, either in high school or in an introductory mechanics course. The goal is for the students to develope an insight into why nuclear physics is of fundamental interest and how the nature of the field determines the character of the experimental equipment and techniques used.

An essential element in modern nuclear physics research is the use of computers in data analysis. This use ranges from programs to extract and display interesting results from voluminous experimental data sets to the use of Monte Carlo procedures to simulate complex processes. Students will eventually gain facility in the use of computers for these purposes. They will of necessity also learn computer languages of common use in current nuclear physics research, including FORTRAN, C, and C++.

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Chapter 1

What is Nuclear Physics?

We consider in our first lecture what the consitutents are that make up our universe. What role in the grand composition of the universe does the atomic nucleus fulfill? We also need to determine the basic parameters of the nucleus, such as its size, its mass, its composition, and any peculiar features it possesses.

1.1 The Constituents of the Universe

We may be convinced by the objects we see around us that the Universe is made of the same stuff as us. But it turns out that the stuff with which we are familiar is only a small piece of the total stuff out of which the Universe is made. In introductory mechanics courses we learn about a physical property called mass. The definition of mass was purposefully vague, because, in fact, we do not really know what mass is. The definition we used was an operational definition, that is, how do we measure mass, not what it truly is. Thus, one establishes a standard unit of mass somewhere and compares all other masses to the standard.

In the twentieth century astronomers discovered that we must broaden our definition of mass to include energy as another manifestation of mass, or perhaps we could say this the other way around, namely, mass is another manifestation of energy. On the grand scale of the Cosmos, Einstein and others showed that all forms of energy have to be included in the calculation of the geometry of the Universe. Starting in the 1930s astronomers discovered that there was substantial mass in galaxies and between galaxy clusters which was not luminous. This is called dark matter. Perhaps even more startling was the evidence that started coming in during the 1990s that there was a huge but dilute component of the mass-energy density of the Universe which basically acts like antigravity. This is called dark energy.

Type of mass/energy	Likely Composition	approximate fraction of total mass/energy density
visible baryonic matter	composed mainly of protons, neutrons, electrons	0.01
baryonic dark matter	ordinary matter that is too dim to see	0.05
nonbaryonic dark matter	exotic particles, axions, massive neutrinos, WIMPS	0.30
dark energy	energy of empty space, quintessence field	0.64

Table 1.1: Contents of the Universe [1]

From Table 1.1 we see that at most about 6% of the Universe is made of ordinary stuff. This is the stuff out of which we are made and everything else is made that is discussed in classical physics! Most of the Universe , 94% is made of stuff for which we have no clear idea. Our concern in these notes is with ordinary baryonic matter. This is the stuff out of which the nucleus is made. Note the irony in our terminlogy. In reality we should be calling dark energy the ordinary stuff. Baryonic matter, after all, is a rather small fraction of the Universe's energy content.

1.2 Examples of Baryonic Matter

A neutral atom contains a positively charged nucleus at its core surrounded by a cloud of negatively charged electrons which gives the atom a total electric charge of zero. Most of the mass of the atom is contained in the nucleus. Electrons are very light compared to the nuclear components. The main components of the nucleus are protons and neutrons. These latter components are the baryonic components. The electrons are actually leptons, which is a different family of particles. Because almost all of the mass of the atom is in the nucleus we refer to the atoms as examples of baryonic matter.

Table 1.2 lists the main parts of baryonic and leptonic matter, i.e., the constituents of ordinary matter. If we were to judge the distribution of elements based on terrestrial abundances we would be grossly misled about the chemical abundances in the universe as a whole. Most of the visible mass, about 75%, is contained in the hydrogen atom, that is, in the proton. Nearly 25% of the visible mass is contained in the mass 4 helium isotope, ^4He, a nucleus containing 2 protons and 2 neutrons. The neutral helium atom has two electrons

Family	Particle	Mass kilograms	Charge Coulombs
Baryon	proton	1.6725×10^{-27}	$+1.602 \times 10^{-19}$
	neutron	1.6748×10^{-27}	0
Lepton	electron	9.1091×10^{-31}	-1.602×10^{-19}

Table 1.2: Masses of Baryons and Leptons

in orbit about the nucleus. A general nucleus of mass A is composed of Z protons and N neutrons, where A = Z + N. The neutral atom has Z electrons in orbit around the nucleus.

Element	number of atoms relative to hydrogen
H	1.
He	0.096
Li	2.3^{-9}
C	2.9×10^{-4}
N	8.0×10^{-5}
O	5.8×10^{-4}
Al	3.5×10^{-6}
Si	4.1×10^{-5}
Ca	2.6×10^{-6}
Fe	3.4×10^{-5}
I	4.1×10^{-11}
Pb	1.3×10^{-10}

Table 1.3: Selected Recommended Elemental Abundances in the Solar System [2]

The other elements in the Periodic Table are quite a small fraction of the baryonic matter of the Universe. In fact, protons and helium nuclei were the overwhelmingly abundant nuclei produced in the Big Bang. Almost all the rest of the chemical elements were created subsequent to the Big Bang in a process called stellar nucleosynthesis. Stars in the latter part of their lives eject the elements they created into space by energetic and sometimes explosive processes. These elements become part of large interstellar clouds which condense into new stars and perhaps also planets around those stars. Some elemental abundances relative to hydrogen for the solar system are displayed in Table 1.3.

Nuclei are one example of baryonic matter, however, there are gigantic conglomerations of baryons in certain stars, called neutron stars. These are the remnants of very massive stars which ended their nucleosynthesis in a violent and disruptive supernova event. Nuclear physics studies of ordinary nuclei in the lead region of the Periodic Table can shed light on the structure of neutron stars.

1.3 Masses and Sizes of Nuclei

The typical atomic dimension is measured in units of Angstroms, 1 Å$= 10^{-10}m$. The typical nuclear dimension is meaured in units of femtometers or Fermis, 1 F$= 10^{-15}m$. Thus, the nucleus is many orders of magnitude smaller than the atom. The nucleus can be imagined to be a sphere, inside of which are bound the protons, neutrons, and other constituents of the nucleus. A handy measure of the radius of the nucleus is $R = 1.2A^{1/3}F$, where A is the atomic mass, $A = Z + N$. Most of the volume of an ordinary piece of matter, a lead brick, for example, is occupied by very low density matter, i.e., the electron clouds around the nuclei. The density of lead is $11.35g/cm^3$, however, the density of the lead nucleus is about $1.45 \times 10^{14}g/cm^3$. The masses of nuclei are more conveniently measured in units of energy. This is a consequence of the Einstein relationship between mass and energy, $E = mc^2$. The unit of energy is called the electron Volt.

$1eV$	$1.602 \times 10^{-19}J$
$1keV$	$1.602 \times 10^{-16}J$
$1MeV$	$1.602 \times 10^{-13}J$
$1GeV$	$1.602 \times 10^{-10}J$

Table 1.4: Energy units Equivalences

Table 1.4 lists the equivalence between electron Volts and the SI unit, Joule. The proton's mass in electron Volts is about 938 MeV = 0.938 GeV. The electron's mass is about 0.511 MeV. The energy required to liberate the electron from the hydrogen atom is 13.6 eV, whereas the energy required to liberate a proton or neutron from a nucleus is usually about 8 MeV.

1.4 What are the Fundamental Forces in Nature?

Recall from introductory mechanics the distinction that was made between kinematics and dynamics. Kinematics is the discipline whereby we describe motion. It basically establishes a language by which we describe the movement of bodies. The study of the the causes of motion is call dynamics, and it is here that the concept of force is introduced. A synonymous word for force is interaction. These two terms will be used interchangeably. At moderate temperatures and energies (this means even in the interior of hot massive stars) we find that there are four fundamental forces. These forces are fundamental in the sense that all interactions between bodies can be ascribed to them. For example, a spring attached to a block exerts a force proportional to the compression of the spring, but on the atomic level we can trace the spring's force to the electromagnetic interaction between atoms which resists bringing atoms closer together than some favored equilibrium separation. The physics terminology is that the spring force $\vec{F} = -kx\hat{x}$ is an 'effective force', as distinct from the underlying fundamental electromagnetic force. The force responsible for large scale motions such as seen in the solar system or of galaxies is the gravitational force. The electromagnetic and gravitational interactions were recognized by the classical physicists and were the only fundamental forces known up to about the 1890s when radioactivity was discovered. The force responsible for much of the radioactive decay (but not all) is called the weak force. An example of radioactive decay by the weak force is the decay of ^{14}C to ^{14}N, which is used in archaeological dating. The discovery of the atomic nucleus in the early 20th century revealed the necessity of a new force, called the strong force, in order to hold the nucleus together. Once it was understood that the positively charged protons were being confined to a very small volume in the nucleus physicist realized that there had to be a strong attractive force of short range between the nucleons (nucleons are protons or neutrons) to overcome the strong electrical repulsion of the protons. Our current understanding of the strong force between nucleons is that it really is an effective force (compare to the discussion of the spring) and the true fundamental strong force is called a 'color force' (Note: This is only terminology. There is no such thing as color as we understand it on the macroscopic scale in the nucleon.) between the constituents of the nucleon. Protons and neutrons themselves are made of smaller particles called quarks. The color force between quarks holds the nucleons together. The effect of the color force spills outside the boundary of the nucleon and so two nucleons interact with each other by the short range effusion of the color force. The theory of the strong color force is called Quantum Chromodynamics, or, QCD. Table 1.5 summarizes the four fundamental forces.

Force	Manifestation
Gravity	holds together large scale systems like galaxies, etc.
Electromagnetism	holds together solid macroscopic bodies
Weak	radioactive decay of certain nuclei
Strong - QCD	holds together nucleons and nuclei

Table 1.5: Fundamental Forces of Nature

Chapter 2

What Experimental Techniques are Used for Nuclear Physics Research?

The study of any physical system requires experimental techniques that are developed with the basic properties of the system in mind. Having established the sizes, masses, and forces in the nucleus we now determine suitable experimental techniques.

2.1 What is meant by Nucleon/Nuclear structure?

Nuclei and nucleons are extended objects. They have finite sizes and therefore we can attempt to measure how the mass, charge, magnetization, or other properties are distributed in space. Bombardment of nuclear or nucleonic targets by probe particles is a very common way to measure nuclear structure. A probe particle interacts with the target nucleus via one or more of the four forces discussed in the previous chapter. In Figure 2.1 theory predicts certain nuclear motion for the nucleons(the small circles). The nucleons are largely confined inside the nuclear volume. Their wave functions (important functions which are determined by the methods of quantum mechanics rather than classical mechanics) extend a little beyond the nuclear surface. If the nucleus is probed using the strong force (a hadronic probe) then the probe has very little probabilty of getting into the center of the nucleus. The strong interaction is large enough that the nucleus is opaque. The information we obtain is mostly about the wave functions in the surface region. If the nucleus is probed with the electromagnetic force (such as by electrons or photons) the strength of the interaction is small enough that much of the interior can be directly seen by the probe. Moreover, the electromagnetic interaction is known to high precision and this helps in interpreting the results from electromagnetic interactions. If the nucleus is probed by the weak force (such as can be done by using neutrinos and recently electrons) the nucleus is practically transparent. The weak force is so weak that very few selected properties of nuclei can be mapped out in

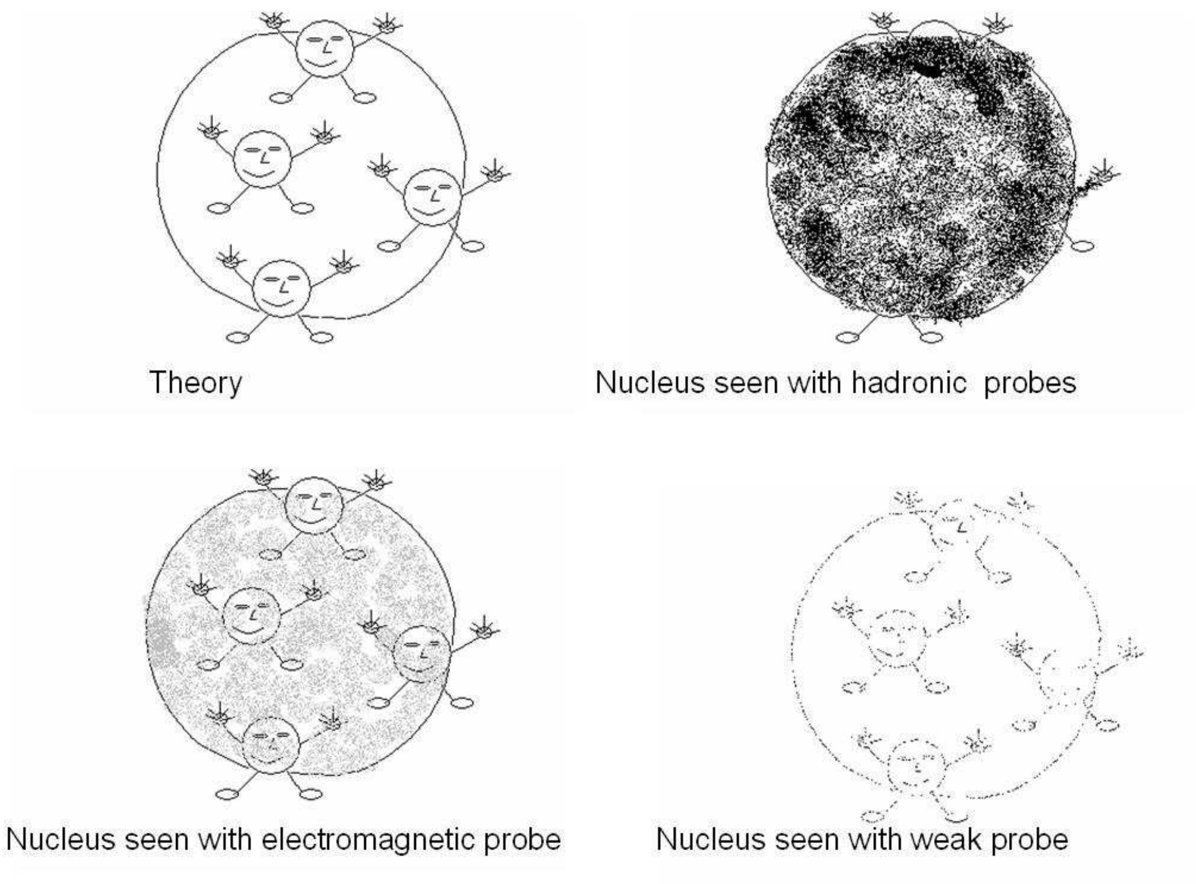

Figure 2.1: The nucleus as seen by theory and three fundamental forces.

a finite experiment time. The gravitational force is so weak that its effects are negligible in nuclear or nucleon structure considerations. The electromagnetic interaction is then a very good probe for studying nuclei and nucleons.

How do we measure such small structures as the nucleus? One way to measure the spatial distribution of matter in the nucleus is by hypothesizing a particular distribution of matter and assuming a force whose value as a function of distance and other parameters are known. One has the initial bombarding energy and momentum of the probe under control. Knowing the initial conditions and using the assumptions about the force and matter distribution mentioned above leads to certain predictions for how particles produced in the bombardment of a target will be distributed in angle and momentum. Recall from elementary classical mechanics that another definition of force is $\vec{F} = d\vec{P}/dt$. So over a given period of time dt a force $\vec{F}$ imparts an impulse $d\vec{P}$. In practice then the nuclear experimentalist measures momentum changes and from this information, using a theoretical model, deduces the underlying nuclear structure which can account for the observed momenta of the reaction products.

2.2 Our Friends the Conservation Laws and Relativistic Kinematics

The conservation laws we studied in elementary classical mechanics must be applied in studies of nuclear reactions. These laws tell us that if there are no external forces acting on a system then the total energy, the total momentum, and the total angular momentum are constant. Namely,

The total momentum of a system: $d\vec{P}_{cm}/dt = \sum \vec{F}_{ext}$
The total energy of a system: $dE_{tot}/dt = \sum_i T_i$, where T_i is an energy transfer mechanism
The total angular momentum of a system: $d\vec{L}/dt = \sum \vec{\tau}_{ext}$, where $\vec{\tau}$ is torque.

The great utility of the conservation laws is that they are true no matter what the forces are that are acting inside the system. If we take as our system the target nucleus and the bombarding projectile, then the total momentum, energy and angular momentum in the final state is equal to the corresponding quantities in the initial state. This is true if we include all the reaction products in the final state. In Figure 2.2 particles a and b collide. A reaction occurs, the details of which do not concern us at the moment. Particles 1, 2, and 3 emerge. The conservation laws guarantee that the total energy, momentum and angular momentum going into the box equals that coming out:

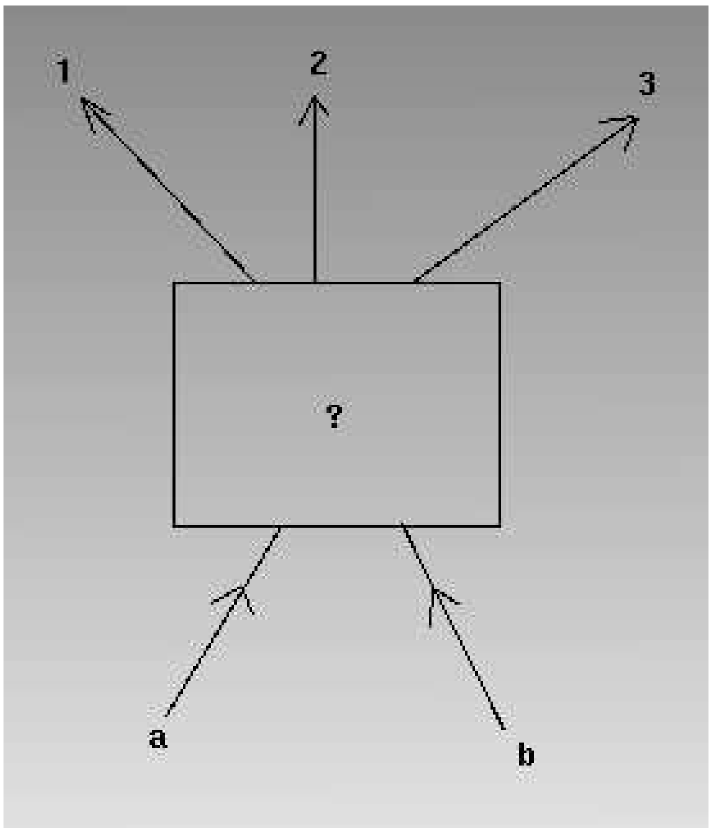

Figure 2.2: Particles a and b collide. The square box with the question mark indicates that some sort of forces are acting within the system, which can be treated as a black box. Particles 1, 2, and 3 emerge. The conservation laws tell us that the total energy, momentum and angular momentum is conserved.

$$E_a + E_b = E_1 + E_2 + E_3$$

$$\vec{P}_a + \vec{P}_b = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

$$\vec{L}_a + \vec{L}_b = \vec{L}_1 + \vec{L}_2 + \vec{L}_3$$

In most experiments energy and momentum are the measured quantities. Measurement of angular momentum is not straight forward since this quantity depends on the origin of the coordinate system, which is only crudely determined in a nuclear physics experiment.

Although the conservations laws are true at all energies we must redefine what we mean by the kinetic energy and the total energy of a particle moving in a force free region at high speeds. In our electron scattering experiments the particles have such high energies that it is essential to treat them using relativistic kinematics. The total energy of a particle includes its mass and energy associated with its momentum. The result for the total energy of a particle in a force free region is:

$E = \sqrt{(mc^2)^2 + (pc)^2}$, where m is the particle's mass, p is its momentum and $c = 3 \times 10^8 m/s$ is the speed of light.

The kinetic energy, K, of the particle is the difference between its total energy and its rest mass energy:

$$K = E - mc^2.$$

It is common practice to measure momentum in MeV/c or GeV/c. The energy equivalents for the electron and nucleon masses were given in chapter 1. Thus a proton which has a momentum of $1GeV/c$ has a total energy of $E = \sqrt{(0.938GeV)^2 + (1GeV/c * c)^2} = 1.371GeV$. Its kinetic energy is $1.371GeV - 0.938GeV = 0.433GeV = 433MeV$.

Rather than carry around the factor c in every calculation it is often not written explicitly, but assumed to be present. This means that for the energy of a particle you might see the expression $E = \sqrt{m^2 + p^2}$. Here it is assumed that the mass is measure in energy units such as MeV and the momentum is measured in momentum units MeV/c. You can always recover the complete expression by replacing every m by $m \rightarrow mc^2$ and every p by $p \rightarrow pc$ consistently.

In Figure 2.2 we have three particles in the final state. We could have more or fewer particles in the final state. We say that Figure 2.2 represents 3 body kinematics. If there were two bodies in the final state, they don't have to be the same as those that went into the box, we would call it a 2 body reaction. Using the conservation of energy and momentum laws we can calculate the complete kinematic variables of the final state particles given a subset of the information.

2.2.1 An Example of 2 Body Kinematics

Consider a special case of Figure 2.2 where we have only two particles emerging, say particles 1 and 2. The reaction is $a + b \to 1 + 2$. The masses of the particles are m_a, m_b, m_1, m_2. Suppose we define the z direction in a Cartesian coordinate system as the direction of the momentum of particle a, i.e., $\hat{z} = \hat{p_a}$ and also consider the case where particle b is a stationary target, $p_b = 0$. Particle 1 emerges from the reaction at an angle θ_1 with respect to the z axis. This information is enough to determine the momenta of particles 1 and 2 in the final state.

conservation of energy: $E_i = E_a + E_b = \sqrt{m_a^2 + p_a^2} + m_b = E_1 + E_2 = \sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2}$.
conservation of momentum: $\vec{p_a} = \vec{p_1} + \vec{p_2}$.

The solution of the kinematics problem is just an exercise in algebra. We will assume we detect particle 1 at angle θ_1 and substitue for the momentum of particle 2, $\vec{p_2} = \vec{p_a} - \vec{p_1}$. The factor p_2^2 in the energy equation becomes:

$p_2^2 = \vec{p_2} \cdot \vec{p_2} = (\vec{p_a} - \vec{p_1}) \cdot (\vec{p_a} - \vec{p_1}) = p_a^2 + p_1^2 - 2p_a p_1 \cos(\theta_1)$.

$E_i - \sqrt{m_1^2 + p_1^2} = \sqrt{m_2^2 + p_a^2 + p_1^2 - 2p_a p_1 \cos(\theta_1)}$. Square both sides of this equation.

After some cancellations you get

$2E_i\sqrt{m_1^2 + p_1^2} = 2p_1 p_a \cos(\theta_1) - (m_2^2 - m_1^2 - E_i^2)$.

This equation can be solved for p_1 and using the conservation laws again one can find p_2 and θ_2. A fortran program for two body kinematics is in Appendix A. Online calculations of two body and three body kinematics are on the CSULA nuclear physics web site,
http://www.calstatela.edu/academic/nuclear_physics/welcome_np.htm.

2.2.2 Three Momentum and Four Momentum Transfer

When electrons interact with an atomic nucleus there are very many possible outcomes. However, despite the fact that there may be many particles emerging from the reaction in Figure 2.2 almost invariably the incident electron also emerges from the reaction. This electron has undergone a change in energy and momentum. Frequently used notation for this is

$\vec{q} = \vec{p} - \vec{p'}$, called the three momentum transfer,

and $\omega = E - E'$, called the energy transfer.

In this notation the unprimed quantities are for the incident electron and the primed quantities are for the electron coming out of the reaction. The four momentum transfer is defined using relativistic notation,

$q_\mu = (\omega, \vec{q})$, which gives an important quantity called $Q^2 = q^2 - \omega^2$.

Units for 3 momentum transfer are GeV/c and for 4 momentum transfer the units are GeV. It is also common to see three momentum expressed in inverse femtometers(F^{-1}) and Q^2 to be expressed in inverse femtometers squared (F^{-2}) or GeV^2. Expressing momenta in inverse units of distance is a consequence of the quantum mechanical relationship between momentum and 'wave number', k, namely, $\vec{p} = \hbar\vec{k}$, where $\hbar$ is Planck's constant divided by 2π. A useful number to remember is $\hbar c = 197.3 MeV \cdot F$.

2.3 What is a Cross Section?

Cross sections are important quantites in nuclear physics. Experiment frequently measures cross sections and theory makes predictions for cross sections. There are several different kinds of cross sections which provide different pieces of information. In order to understand these important quantities we will first consider the 'total' cross section. The nuclear target is very small and the range of the nuclear force does not extend much beyond the nucleus. We need to set up a macroscopic target, shoot probe particles at this target and wait for a chance encounter between a probe particle and a target nucleus. This nucleus could be anywhere within the footprint of the incident beam of probe particles. Consider Figure 2.3 in which the incident probe particles (projectiles) impinge on a target of area S. Inside the macroscopic target there are nuclear targets of area $\delta\sigma$. The probability, dp_1, for a single probe to hit the nuclear target is just the ratio of areas.

$dp_1 = \delta\sigma / S.$

If there are dN_0 incident probe particles the number, dn_1, of them which will hit the nuclear target is

$dn_1 = dN_0 dp_1 = dN_0 \delta\sigma / S.$

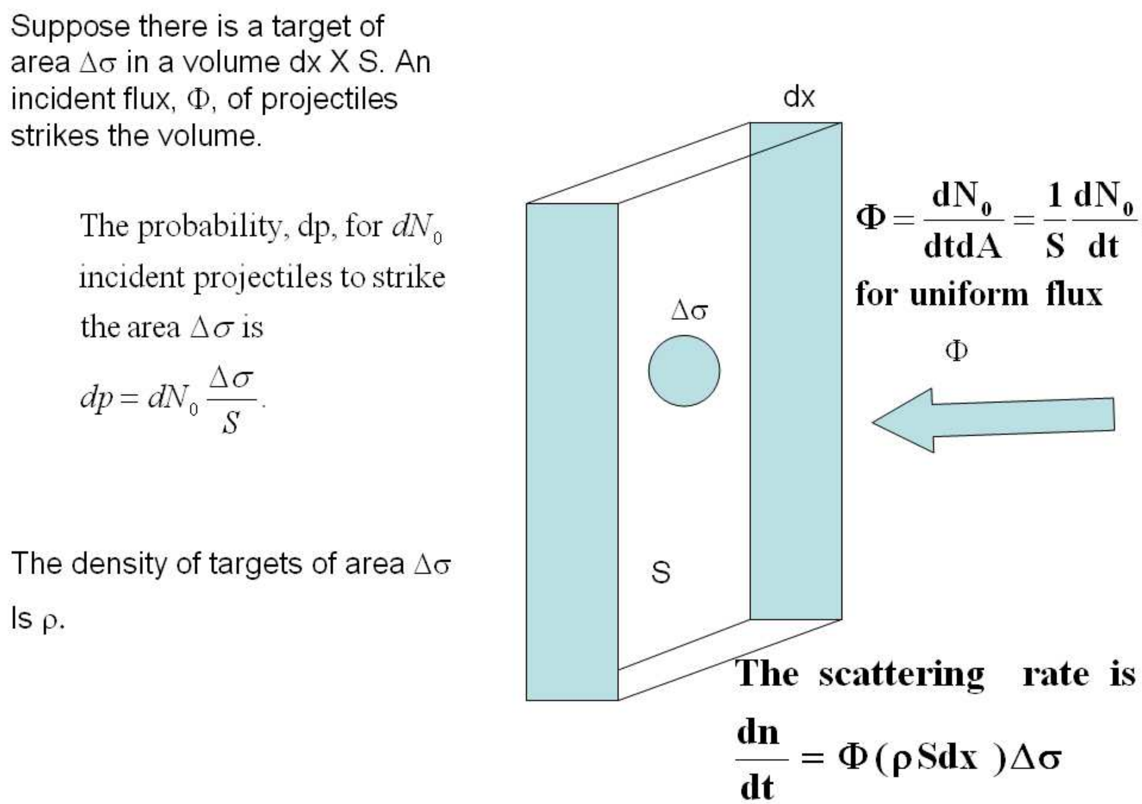

Figure 2.3: Total Cross Section.

Suppose the density of nuclear targets is ρ. For a macroscopic target of area S and thickness dx, the number of collisions, dn, will increase in proportion to the number of targets, $dN_{targets} = \rho S dx$.

$$dn = dN_0(\delta\sigma/S)\rho S dx = dN_0 \delta\sigma \rho dx.$$

If the measurement occurs over a time dt, then the total reaction rate, dn/dt is given by

$$dn/dt = (\delta\sigma)(\rho dx) dN_0/dt.$$

If the incident beam probe particles have an electrical charge q, then $dN_0/dt = I/q$, where I is the current. At the electron accelerator the current I can be any value from a few nanoamps to 200 microamps. For a $10\mu A$ electron beam the incident electron rate is

$$dN_0/dt = 10 \times 10^{-6} Coulomb/sec/1.6 \times 10^{-19} Coulomb = 6.25 \times 10^{13} electrons/sec.$$

Suppose we have a target of lead, which has a mass density of $11.35 g/cm^3$. We use Avogadro's number, $N_A = 6.02 \times 10^{23}$ $atoms/mole$, the atomic mass of lead, $M_A = 207.2$ $g/mole$, to determine that the density of lead nuclei($nuclei/cm^3$), ρ, is

$$\rho = (6.02 \times 10^{23} atoms/mole)(11.35 g/cm^3)/(207.2 g/mole) = 3.3 \times 10^{22} atoms/cm^3.$$

Now suppose our target is a thin foil of lead of thickness $0.2mm = 0.02cm$. The reaction rate we expect is

$$dn/dt = (6.25 \times 10^{13}/sec)(3.3 \times 10^{22}/cm^3)(0.02cm)(\delta\sigma).$$

$$dn/dt = 4 \times 10^{34}/(cm^2 \cdot sec) \times (\delta\sigma).$$

Notice that the units for the cross section, $\delta\sigma$, must be in cm^2 in order that the rate be given in inverse time. A typical cross section for electron scattering might be $1F^2 = 1 \times 10^{-26} cm^2$, which gives a rate of $4 \times 10^8/sec$. This example deals with a total cross section. Actually it is much more likely to measure finer aspects of the reaction, such as how many particles come out of the reaction with given energies and given angles. These more detailed measurements give us 'differential' cross sections. For example, we might measure the elastic scattering cross section for electrons of 1 GeV incident energy on lead. The differential elastic scattering cross section is denoted by $d\sigma/d\Omega$, where $d\Omega$ is a differential element of solid angle. The solid angle is the fraction of a sphere's total area that a differentially small area dA covers when this area dA is totally within the surface of the sphere. That is we can form a comparisons of ratios. Suppose we have a small area dA on the surface of a sphere of radius

R. The solid angle $d\Omega$ subtended by the area dA is given by

$d\Omega/(4\pi) = dA/(4\pi R^2)$, or $d\Omega = dA/R^2$.

The elastic scattering cross section is a function of the angle, θ at which we observe the scattered electrons and of the incident electron energy for a given target, $d\sigma(\theta)/d\Omega$. We would place a detector of a given solid angle $\Delta\Omega$ at an angle θ. The count rate in the detector for this finite, but small, solid angle is

$dn/dt = (I/q)(\rho dx)(d\sigma(\theta)/d\Omega) \cdot \Delta\Omega$.

In this case we see the differential cross section has units of area/(solid angle). The unit of solid angle is called the steradian, abbreviated as 'sr'. The sphere subtends a solid angle of $4\pi\ sr$. A hemisphere subtends a solid angle of $2\pi\ sr$.

2.4 Experimental Equipment used in Hall A at Jefferson Lab

Nuclear and Particle physics relies heavily on the fact that a charged particle moving through a material produces ionization. Ionization occurs when electrically neutral atoms are broken into heavy ions and liberated electrons. The so called free charge can be collected by the application of an electric field. In another technique the exicted atoms or molecules deexcite by emitting visible or near visible light as photons. Neutral particles, which by themslves do not produce a trail of ionized atoms, are rendered visible by a secondary process. The neutral particles can interact with atomic nuclei and produced energetic charged secondary particles. The charged secondaries are then detected by the ionization they produce. In the case of photons, which are themselves uncharged, electrons in the material can gain a momentum impulse from the interaction with the photons. These electrons, or the atomic photons from the excited atoms, are subsequently detected.

An important technique, especially for particles with kinetic energies greater than around 100 MeV, is trajectory manipulation of charged particles by the use of magnetic fields. At Jefferson Lab, for example, significant detector components are: scintillators, Cerenkov detectors, wire chambers, shower counters, neutron detectors, and large magnetic devices such as the Hall A high resolution spectrometers(called HRS for short).

2.4.1 Ionization Techniques

Wire chambers are an example of detectors that rely on the ionization trail left by a charged particle. These devices are used to determine the position and direction of motion of charged particles. Details of the vertical drift chambers (VDC) used at JLab can be found at http://www.jlab.org/ fissum/vdcs/documentation/docs.html.

VDCs are low mass detectors. This means that charged particles entering them encounter only a small amount of $mass/cm^2$ and at JLab the common use requires that the charged particle sails right through these detectors with very little energy loss. A special gas mixture is held between two high voltage planes, typically -4kV. In the middle of the chamber, thin wires are strung running parallel to the high voltage planes, see Figure 2.4. These wires are held at ground potential (0 Volts). There are 400 wires with a wire spacing of 6mm. A fast charged particle creates a trail of free electrons along its ionization path. Because of the electric field between the wires and the high voltage planes the electrons drift towards the wires. The drift velocities of the electrons in the gas is about 51 $\mu m/ns$ ($1\ ns = 1 \times 10^-9\ s$).

By measuring the amount of time it takes for an electron to reach the sense wire, where its acquisition is detected by the VDC electronics, one can determine how far from the wire the ionization trail started. A plot of the electron drift time is shown in Figure 2.5. In this figure TDC stands for Time to Digital Conversion. The TDC is a fast clock which has a start pulse and stop pulse input. The time difference between the start and stop pulses can be less than a nanosecond to microseconds. It is a common electronic component in nuclear physics data acquisition system.

Ideally 5 sense wires are triggered by the passage of a single fast charged particle. Within each VDC there are actually two sets of wires forming the 'U' plane and 'V' plane. The U and V planes are at right angles to each other. There are two VDC packages used in the Hall A HRS detector huts. We can determine the location of the hits in each VDC to within about 125 μm and using this information from both VDCs enables a good measure of the trajectory of the fast charged particle to be determined. This is shown in Figure 2.6.

The VDCs are located in the shielded detector huts on top of the HRS. They are in a region free of the spectrometer's magnetic fields. Given the VDC information about the impact point and angle of impact in the focal plane of the spectrometers we use software and the known magnetic properties of the HRS to trace the particle's path back to the target. It is the target parameters (Basically we want to know the vector momentum of the particle at the target.) which contain the nuclear physics information.

Notice that in order to get the electron drift time in the VDC we need a start pulse and a stop pulse. How are these pulses generated? If we had a trigger pulse for starting the TDC we could stop the TDC with the pulse from the sense wire which signals the arrival of the drifting electrons. We have devices which can produce a sharp (in time) pulse when a charged particle passes through them. These are the scintillator detectors. The electronics scheme at HALL A is to start the TDC with the sense wire signal and stop the TDC with the scintillator signal. The scintillator signal is delayed in order to accomodate the long drift times of the electrons in the VDC. The scintillator detector is discussed in the next section.

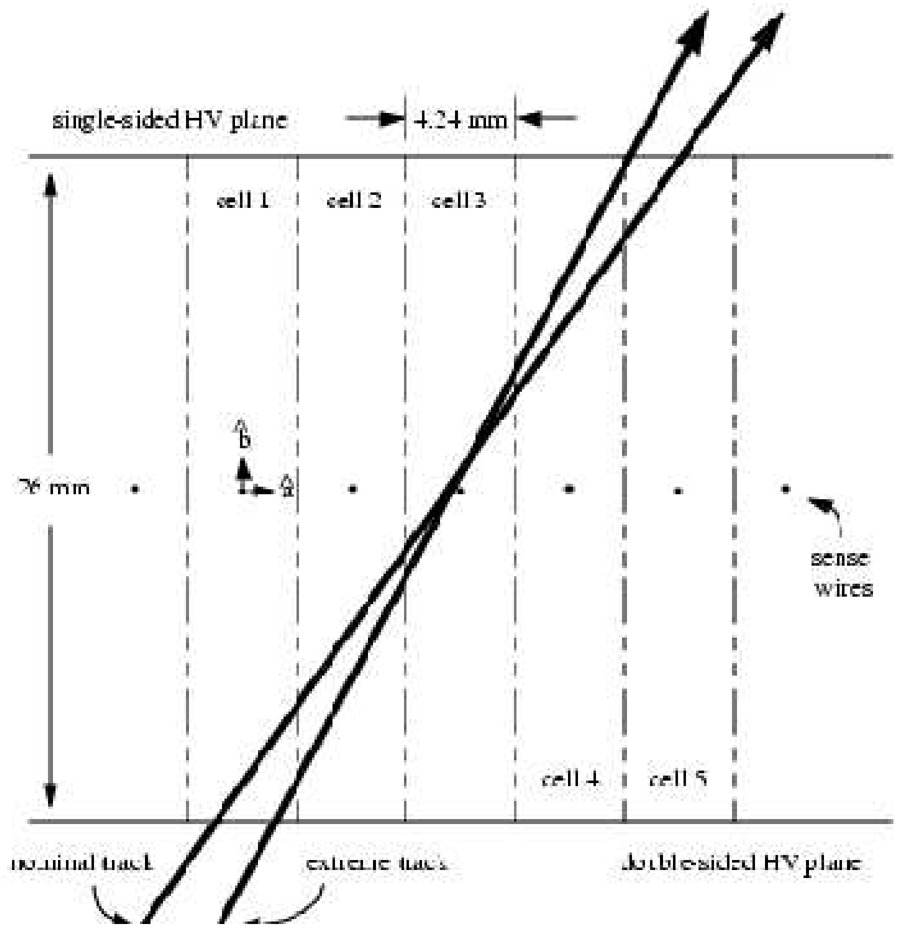

Figure 2.4: The basic elements of the VDC.We see particle trajectories which leave a primary ionization trail. By measuring the time it takes for the electrons in the primary trail to reach the sense wire one can determine the position and angle of the trajectory [3].

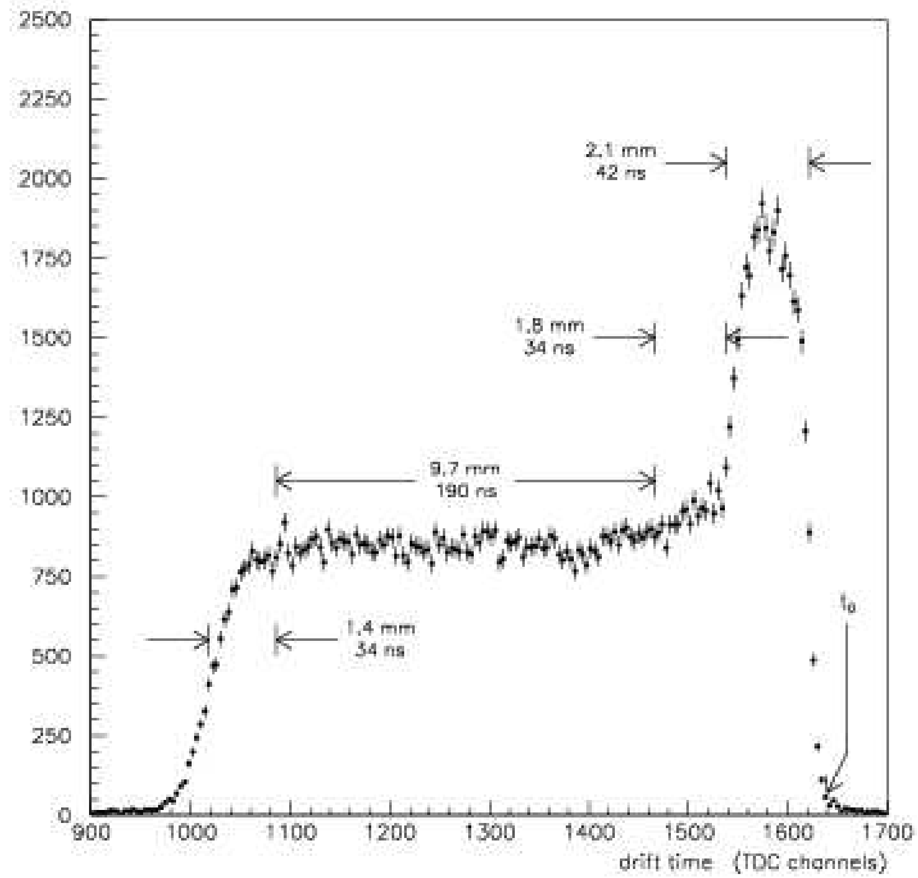

Figure 2.5: The electron drift time spectrum for a single wire. The electronics is set up so that short drift times occur at large TDC values. Time zero, t_0, is at channel 1640.The time bin is 2.0ns. The total time spectrum is about 300 ns [3].

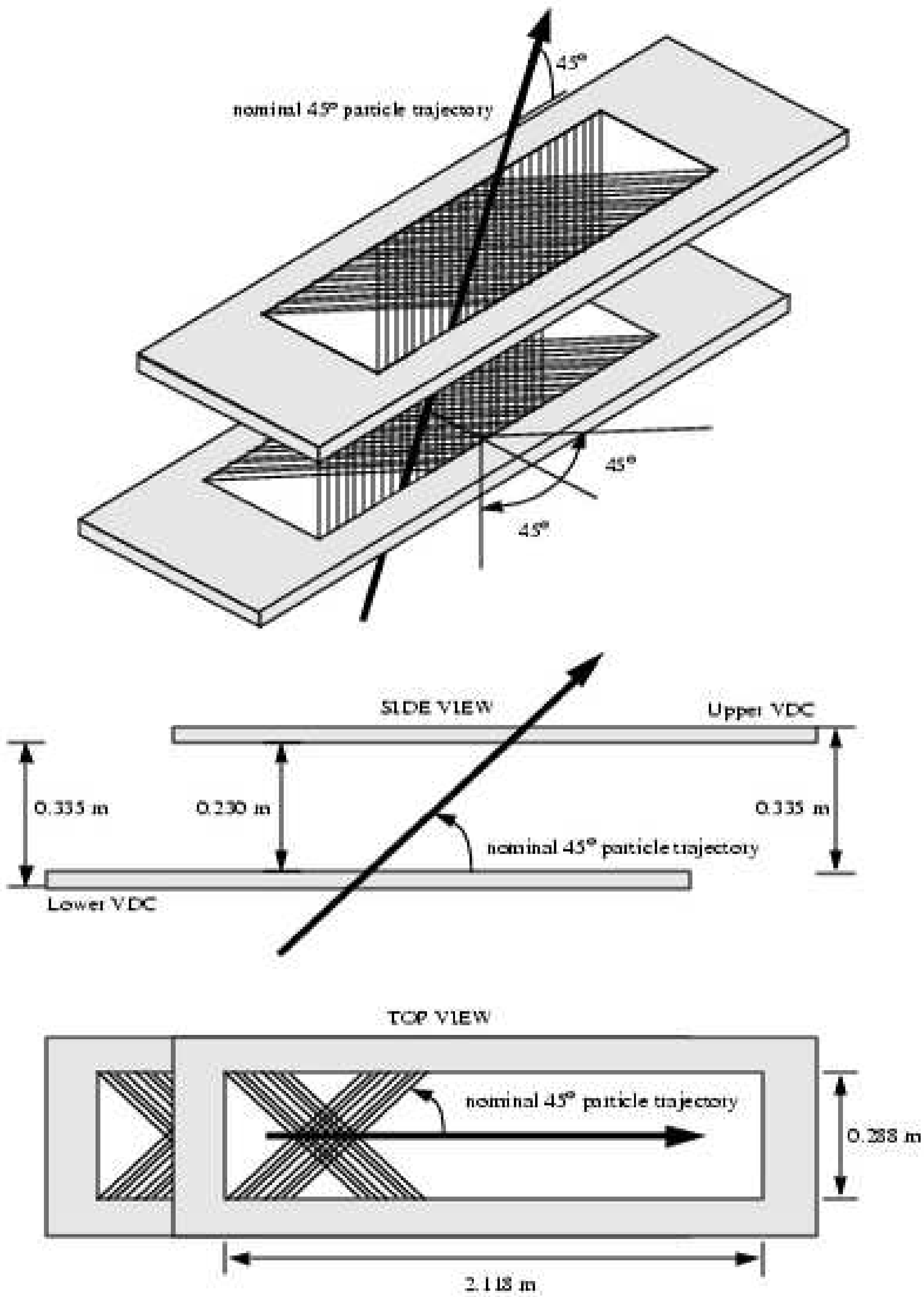

Figure 2.6: The detector packages showing the use of several VDCs separated by a large distance. Knowing the positions of the tracks in the VDCs and the separation of the sense wire planes enables a good determination of the angle of the trajectory to be determined [3].

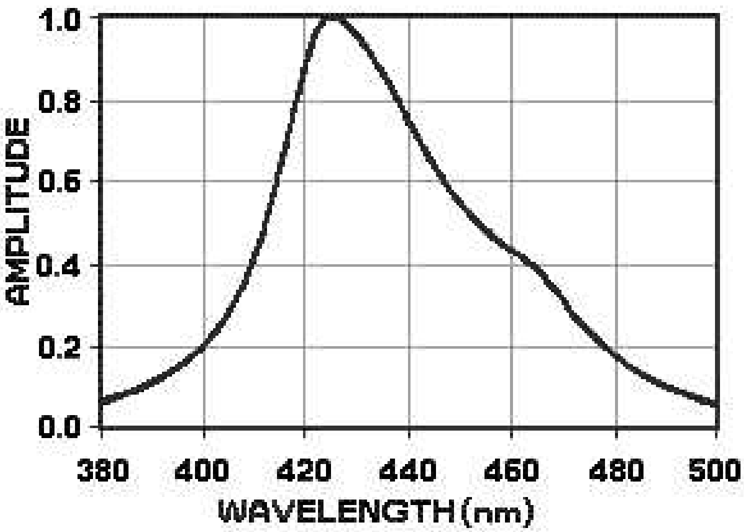

Figure 2.7: The emission spectrum from the scintillator RP408 [4].

2.4.2 Scintillators and Photomultiplier tubes

Scintillators are plastic like materials which emit very short pulses of light when an ionizing particle passes through them. These are compounds of hydrogen and carbon and oxygen (plexiglass $(C_5H_8O_2)_n$ which have been doped with special molecules called wave length shifters(WLS). A charged particle passing through this material excites molecular levels which decay by emitting ultra violet light. Our method of detecting the light requires that the UV be transmitted some distance (About 1 meter is not unusual.). However, there is a strong absorption of the UV light. This is overcome by adding WLS which absorb the UV and emits light at longer wave lengths in the visible. These longer wave length photons can be transported inside light guides to photomultiplier tubes which convert the photons into electrical pulses. An example of the light output from a particular manufacturer's scintillator is in Figure 2.7.

some references
http://rd11.web.cern.ch/RD11/rkb/PH14pp/node166.html
http://www.shef.ac.uk/physics/teaching/phy311/scintillator.html

Chapter 3

Simulation of Experiments and Monte Carlo Techniques

Computers are essential elements of data analysis. Modern nuclear physics experiments have huge bodies of data which can only be dealt with by computers. Experimental equipment is built with certain ideal characteristics in mind but real equipment has its limitations, however. Moreover, the physical steps in transporting a reaction product from the target to the detector can be so involved that a proper understanding of the experimental results can only be achieved by comparing them to simulated data. By seeing how the experimental equipment responds to known simulated data we have a better understanding of the results from experiment.

3.1 Monte Carlo Simulation

In simulating a physical process, such as the history of a particle from its creation at some intial point, say point 0, to its final detection say at point f, we need to know the responses of the system to the particle along its trajectory. For example, a proton created with initial momentum $\vec{p}_0$ inside the target material can undergo a nuclear reaction as it passes out of the target into vacuum, producing a variety of reaction products. It can also simply undergo multiple scatterings which change its momentum vector. In order to simulate the passage of the proton from its point of creation into the vacuum we must know the probabilities for nuclear reactions and for multiple scattering. These probabilities are expressed as a distribution function. Supposing the proton's momentum is known at a position vector $\vec{r}$, we let the proton make a displacement $d\vec{r}$ to a new position $\vec{r}+d\vec{r}$ and ask what will be the new momentum or does the proton suffer a nuclear reaction during the small displacement $d\vec{r}$? The Monte Carlo procedure involves selecting the proton's new condition at $\vec{r}+d\vec{r}$ by using the probability distribution functions along with a random number generator. In the most common context a random number generator is a computer program that generates a random number r, normally such that $0 \leq r \leq 1$ with uniform probability. (Strictly speaking, these numbers generated by a computer algorithm are pseudo random numbers, but let us not worry about this distinction right now.) Let's consider what is required to simulate a function f(x) using the Monte Carlo procedure.

Figure 3.1 shows a function defined for $0 \leq x \leq 10$. In order to simulate this function we need a procedure which will give us back this function after a sufficiently large number of trials. Imagine that we mark up the curve with little boxes, called plaquettes. We can make these boxes as small as is necessary to get the level of accuracy our problem demands. We will label the boxes with number n, $1 \leq n \leq n_{max}$ and also include the value of the x coordinate appropriate for that plaquette number. Cut up the graph into the plaquettes and toss them into a hat. If we now blindly reach into a hat and pull out a plaquette we will read the plaquette number as well as the value of the x coordinate. After recording the value of x we return that plaquette back to the hat and draw again. Each draw is a trial. The upshot of the trial is that we have selected a value of x. If we make a table listing how many times we have picked a particular value for x, after a sufficiently large number of trials the list should simulate the function f(x). The number of plaquettes above a particular value of x is proportional to the value of f(x). If our function had a value of zero for a particular value of x, then there would be no plaquettes to pick for that value of x and it would never appear in our random drawing of plaquettes.

Now, if we restricted ourselves to drawing plaquettes from a hat you might ask what is the purpose of numbering the plaquettes? Indeed, the only useful information we get from the plaquette is the value of x. However, the plaquette numbering scheme we picked is not arbitrary and it leads to a useful numerical procedure well suited to a computer. We take note of the following:

Each plaquette has an area ΔA

and the total area under the curve is approximately $A \approx n_{max} * \Delta A$.

The smaller we make the plaquettes the more closely we approach the true area A. In fact, $n_{max} = A/\Delta A$. By adopting our numbering scheme we realize that the plaquette number now has a useful meaning. The area under the curve from the lowest value of x, x_{lo}, to the value of x written on the plaquette is $A(x) = n * \Delta A = \sum_{i=1}^{n} \Delta A_i$, where $\Delta A_i = \Delta A$. Here is where the random number generator comes into play. The plaquettes all have the same chance of being pulled out of the hat. The random number r has equal chance of being any value between 0 and 1. Hence, we can determine the value of n by setting $n = r * n_{max}$. Now making the transition to calculus as $\Delta A \rightarrow 0$ we write

$n * \Delta A = (r * n_{max}) * \Delta A = r * (n_{max} * \Delta A) = r * A = \int_{x_{lo}}^{x} f(x)dx.$

and $A = \int_{x_{lo}}^{x_{hi}} f(x)dx$. So the value of x is implicitly chosen by the integral equation.

$r = \frac{1}{A}\int_{x_{lo}}^{x} f(x)dx$.

In the graphical interpretation of the simulation procedure we assumed the areas were positive. If the function you want to simulate has negative lobes you take the absolute values of the area to do the simulation.

3.1.1 Example 1, A constant distribution

If $f(x) = c$ for $x_{lo} \leq x \leq x_{hi}$ then $A = \int_{x_{lo}}^{x_{hi}} cdx = c(x_{hi} - x_{lo})$.

$r = \frac{1}{A}\int_{x_{lo}}^{x} cdx = (x - x_{lo})/(x_{hi} - x_{lo})$.

3.1.2 Example 2, An exponential distribution

Here $f(x) = cexp(\lambda x)$ then $A = \int_{x_{lo}}^{x_{hi}} cexp(\lambda x)dx$

$A = \frac{c}{\lambda}(exp(\lambda x_{hi}) - exp(\lambda x_{lo}))$.

$r = \frac{1}{A}\frac{c}{\lambda}(exp(\lambda x) - exp(\lambda x_{lo}))$.

Solving for x, $x = \frac{1}{\lambda}ln(r(exp(\lambda x_{hi}) - exp(\lambda x_{lo})) + exp(\lambda x_{lo}))$.

3.1.3 Example 3, An isotropic distribution in angles

Spherical coordinate systems are often used to describe how particles are scattered into angle. Consider the special case where the angular distribution is uniform, that is, every direction of emission is equally probable. In figure 3.2 the probability that particles exit the sphere through the area element dA is $dP = dA/(4\pi R^2) = d\Omega/(4\pi) = (dx)(d\phi)/(4\pi)$, $x = cos(\theta)$. Here $0 \leq \phi \leq 2\pi$ and $-1 \leq x \leq 1$. In this coordinate system the area dA and the probability dP depend linearly on the azimuthal angle ϕ, so we can simulate ϕ by setting:

$\phi = 2\pi r$, $0 \leq r \leq 1$.

The polar angle θ appears in a function, $cos(\theta)$, however, the variable $x = cos(\theta)$ is linearly dependent on dP, so we simulate $cos(\theta)$ by setting:

$cos(\theta) = 2r - 1$. And then we can solve for θ.

There is a FORTRAN program on cyclone under /home/aniol/src/pickhi1.f which uses the plaquette method. The distribution to be simulated is given as a histogram. We will find that many times the distributions we want to simulate are not given by an analytic formula, so the histogram is a good choice as a model to simulate. In addition, any analytical function can be represented as a histogram as well.

The plaquette method can be extended to distributions depending on more than one variable. The FORTRAN code /home/aniol/src/pickhi2.f simulates a two dimensional histrogram. Sometimes distributions depend on several variables jointly, like angle and momentum for a scattered particle. In this case we can't treat the probabilities for the separate variables as uncorrelated.

In Appendix B there is a short c++ program using Monte Carlo techniques.

3.2 Some Monte Carlo Exercises

We now consider some examples of using Monte Carlo for solving problems. In our usage the common problems require tracking a particle through space. The steps involved are basically three:

1) State the problem. What results do you seek?

2) Establish the geometry of the problem space. There may be several regions.

3) Establish the methods to be used to handle the particle's fate in each of the regions.

3.2.1 Evaluate π

Our first example will be to evaluate pi (π), a common introductory exercise. We start with figure 3.3.

Step 1: We want to know the value of π.

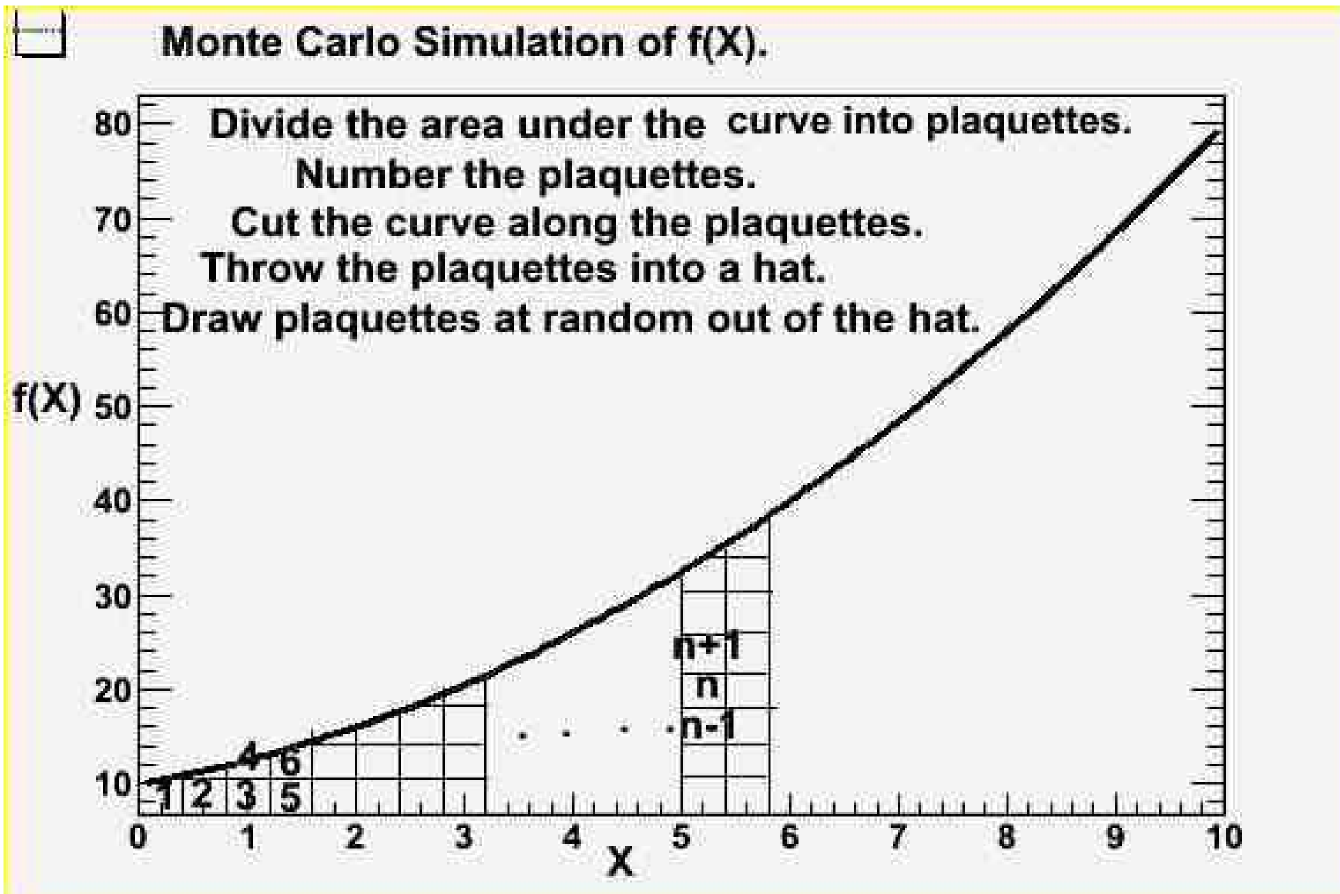

Figure 3.1: Divide the area under the curve into plaquettes numbering them from 1 to N_{max}. Also include on the plaquette the value of x. Cut up the curve and toss the plaquettes into a hat. Randomly drawing the plaquettes from the hat and reading off the x value is a Monte Carlo procedure for simulating the curve.

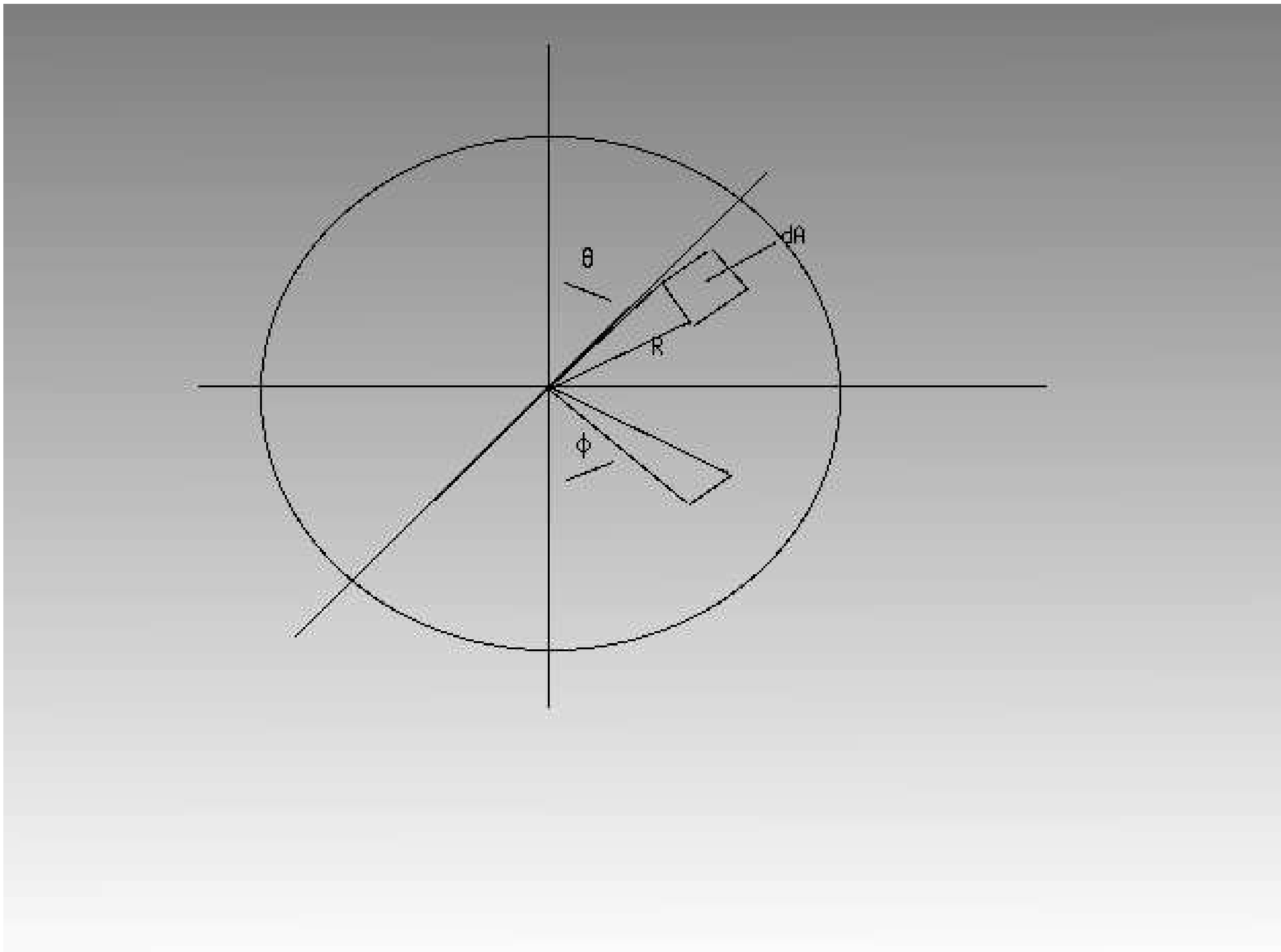

Figure 3.2: A sphere of radius R. Consider the element of area dA located at angles θ and ϕ. From the diagram $dA = (Rsin(\theta)d\phi)(Rd\theta) = R^2 d\phi dx$, where $x = cos(\theta)$. The element of solid angle is $d\Omega = sin(\theta)d\theta d\phi$.

Step 2: Draw a square centered on the origin of the Cartesian coordinate system. The square has dimension 2 x 2. Inscribe a circle of radius R = 1, centered on the origin. We have two geometrical regions here, the area inside the circle and the area inside the square. It is ok that the two areas overlap.

Step 3: The method we use is to select points (x,y) at random inside the square. These points will either be within the circle or outside the circle, but still in the square. If the points are chosen at random inside the square then the probability we get a point inside the circle is $P_{circle} = (Area\ of\ circle)/Area\ of\ square)$ or

$P_{circle} = (\pi R^2)/(2 \times 2) = \pi R^2/4.$

Suppose we select N_{total} trial points and find that we get inside the circle N_{circle} times. Then

$P_{circle} = N_{circle}/N_{total} = \pi R^2/4$. So $\pi = 4N_{circle}/N_{total}$. (Note we picked R = 1).

The way we select the points (x,y) at random is to set $x = 2r_1 - 1$ and $y = 2r_2 - 1$, where the random numbers are $0 \leq r_1 \leq 1$ and $0 \leq r_2 \leq 1$ and come from two separate calls to the random number generator. We must make a separate call for each of the two coordinates, x and y. A common mistake is to use the same random number for both x and y, which introduces a correlation between x and y and gives you a straight line relating x and y instead of a random scattering of the points.

We determine whether or not the point is inside the circle by using the logic functions of the computer languase we are using. Determine the distance the point is away from the origin, $d = \sqrt{x^2 + y2}$. If $d \leq 1$ then increment the counter N_{circle}. After we have run through our N_{total} we can obtain the probability P_{circle}. Write the computer program that gives you π. Do you expect your program to give you the exact value of π ?

3.2.2 A random walk with manholes and Joe's Bar

A drunken man leaves the origin of coordinates. What is the chance that he will make it home? We use figure 3.4.

Step 1: We want to know the probability that he reaches home, P_H, that he falls into the manhole, P_m, or that he stays at Joe's Bar, P_J.

Step 2: We define the geometric regions:

Region 1, R1: is eveywhere outside of the boundaries, $x \leq 0$ or $x \geq x_W$ or $y \leq 0$ or $y \geq y_W$.

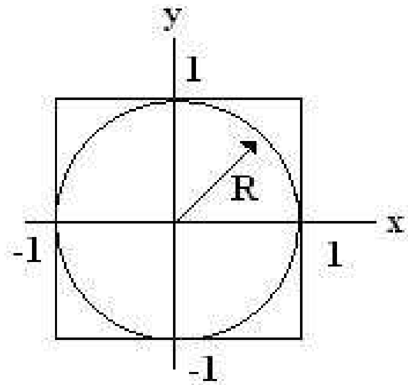

Figure 3.3: A circle of radius R = 1 inscribed inside a square of dimension 2 x 2.

Region 2, R2: This is the manhole, all points (x, y) where $d = \sqrt{(x - x_m)^2 + (y - y_m)^2} \leq R_m$.

Region 3, R3: This is his home, all points for which $(x_H \leq x \leq x_W)$ and $(y_H \leq y \leq y_H)$.

Region 4, R4: Joe's Bar, all points for which $(x_1 \leq x \leq x_2)$ and $(y_1 \leq y \leq y_2)$.

Region 5, R5: All the rest of the space.

Step 3: What methods should be used in these regions?

R1: If a step takes him outside the boundary, select another random step.

R2: If the step ends in the manhole increment the manhole counter N_m.

R3: If the step ends at home increment the home counter N_H.

R4: If the step ends in Joe's Bar decide first if he is to stay. This can be done by chosing a random number r. If $r \leq 0.5$ then he stays and increment the Joe counter N_J. If $r > 0.5$ then Joe takes him to a door, a, b, c, or d. We can assign a probability to each door and choose a random number, r. Suppose we pick door a if $0 < r < 0.25$, door b if $0.25 < r < 0.50$, door c if $0.50 < r < 0.75$ and door d if $0.75 < r < 1.0$.

R5: Here he takes a random step of constant length s but random direction ϕ, where $\phi = 2\pi r$, and r is the random number. Then $x_s = s * cos(\phi)$ and $y_s = s * sin(\phi)$. His new coordinates (x_{new}, y_{new}) are related to his old coordinates (x_{old}, y_{old}) by $x_{new} = x_{old} + x_s$, $y_{new} = y_{old} + y_s$.

Solve this problem for the following conditions: $s = 0.5$, $x_W = y_W = 10$, $x_H = y_H = 8$, $x_m = 8$, $y_m = 4$, $R_m = 1$, $x_1 = 3$, $y_1 = 6$, $x_2 = 5$, and $y_2 = 8$.

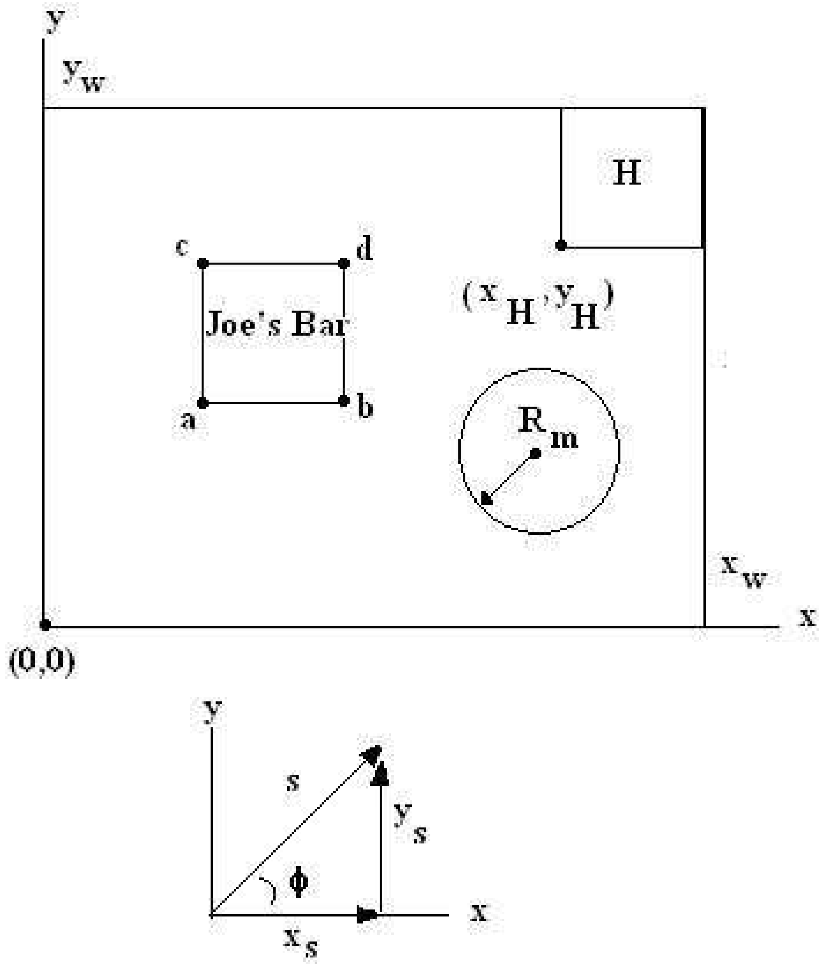

Figure 3.4: A drunk leaves the origin and is confined to the regions within the borders $0 \leq x \leq x_W$ and $0 \leq y \leq y_W$. Along the way to his home located at $x_H \leq x \leq x_W$ and $y_H \leq y \leq y_W$ the man can fall into a manhole of radius R_m centered at (x_m, y_m). He can also wander into Joe's Bar whose four corners are at points a (x_1, y_1), b (x_2, y_1), c (x_1, y_2), and d (x_2, y_2). Once in Joe's Bar he has a probability of 0.50 of staying there. If he leaves, Joe puts him with equal probability at either of the four exits a, b, c, or d. The man takes a step size of constant length, s, but in a random direction, ϕ.

Chapter 4

ROOT, ESPACE, The Hall A analyzers

Nuclear and particle physics experiments generate huge volumes of data, in the terabyte range even. The data are generated by electronic devices and detectors. They may appear as voltage or current pulses or digitally as a measure of time. These data are translated into physics units, like angles, momenta, or energy. The programs that take the raw data and produce the physically interesting quantities are ESPACE and the Hall A analyzer built on the ROOT system. ESPACE and the Hall A analyzer are examples of data analysis programs. The general flow of information in a nuclear physics experiment:

1) take the signals from the detectors using a data acquisition program (DAQ) which writes the raw data onto mass storage devices
2) data analysis programs take the raw data off the mass storage devices, convert the raw data into physically relevant quantities and produce histograms or graphs of interesting quantities.

Our principal data analysis program will be the JLab Hall A analyzer. The ROOT system was developed at CERN. Documentation on ROOT is available on cyclone at

`/data/apps/root/4.02.00/docs/Users_Guide_4_04.pdf`

The ROOT package is very general and can also be used simply as a graphics package. We will first be interested in the basics of ROOT, so read Chapter 2, "Getting Started".

Appendix A

Examples of FORTRAN and C++ programs for 2 Body Kinematics

Here is a fortran progam that calculates 2 body kinematics. Use an editor, like emacs or vi, to create the file kin2b.f. Then enter the following commands, assuming you are on a linux platform.

g77 kin2b.f

Assuming no errors were found you then type

mv a.out kin2b.out

To run the program you can type

kin2b.out

or if that doesn't work try

./kin2b.out.

```
*****************************************************************
c kin2b.f - two body relativistic kinematics, a+b→1+2, b at rest
double precision ma, mb, m1, m2, pa, p1, p2,th1,th2,ei,ea,e1,e2,k1
double precision csth1,f,alpha,rad,th1r,th2r,g,csth2,ka,k2
data rad/57.29/
character*1 angle
write(6,*)'two-body kinematics a+b→1+2'
```

```
write(6,*)' momentum x = px, kinetic energy x = kx'
write(6,*)'enter ma,mb,m1,m2,ka in GeV'
read(5,*)ma,mb,m1,m2,ka
ea = ma + ka ! ka = kinetic energy of particle a
ei = ea + mb
pa = dsqrt(ea*ea - ma*ma)
alpha=m2*m2-m1*m1-ei*ei+pa*pa
h=alpha*alpha-4.*ei*ei*m1*m1
100 write(6,*)'enter theta1 in degrees'
read(5,*)th1
th1r=th1/rad
csth1=dcos(th1r)
f=2.*(ei*ei-pa*pa*csth1*csth1)
g=alpha*pa*csth1/f
p1 = -g + dsqrt(h/2./f + g*g)
e1 = dsqrt(m1*m1+p1*p1)
k1 = e1 - m1
write(6,200)p1,th1,k1
200 format('p1 = ',e11.4,' GeV/c,',' theta1 = ',f5.2,' deg, ',
+ 'k1 = ',e11.4,' GeV')
c now find momentum of particle 2
e2 = ei-e1 ! conservation of energy
k2 = e2 - m2
p2 = dsqrt(e2*e2-m2*m2)
csth2=(pa-p1*csth1)/p2 ! uses conservation of momentum
th2r = dacos(csth2)
th2 = rad*th2r
write(6,201)p2,th2,k2
201 format('p2 = ',e11.4,' GeV/c,',' theta2 = ',f5.2,' deg, ',
+ 'k2 = ',e11.4,' GeV')
write(6,*)' do another angle? y/n'
read(5,*)angle
if(angle.eq.'y') then
goto 100
endif
stop
end
************************************************************************
```

Sample output from the 2 body kinematics program.

two-body kinematics a+b→1+2
momentum x = px, kinetic energy x = kx
enter ma,mb,m1,m2,ka in GeV
.511e-3 .938 .511e-3 .938 1.
enter theta1 in degrees
40.
p1 = 0.8007E+00 GeV/c, theta1 = 40.00 deg, k1 = 0.8002E+00 GeV
p2 = 0.6441E+00 GeV/c, theta2 = 53.04 deg, k2 = 0.1998E+00 GeV
do another angle? y/n
y
enter theta1 in degrees
45.
p1 = 0.7623E+00 GeV/c, theta1 = 45.00 deg, k1 = 0.7618E+00 GeV
p2 = 0.7097E+00 GeV/c, theta2 = 49.43 deg, k2 = 0.2382E+00 GeV
do another angle? y/n
n

Here is the same program written in c++. Instead of using g77 to compile the program use g++.

```
// kin2b.cpp - two body kinematics
#include<iostream>
#include<cstdlib>
#include<cmath>
int main()
{
double  ma, mb, m1, m2, pa, p1, p2,th1,th2,ei,ea,e1,e2,k1;
double  csth1,f,alpha,rad(57.29),th1r,th2r,g,csth2,ka,k2,h;
char angle;
std::cout <<"two-body kinematics a+b->1+2"<<'\n';
std::cout <<" momentum x = px, kinetic energy x = kx"<<'\n';
std::cout <<"enter ma,mb,m1,m2,ka in GeV"<<'\n';
std::cin >>ma>>mb>>m1>>m2>>ka;
 ea = ma + ka;   // ka = kinetic energy of particle a
 ei = ea + mb;
 pa = sqrt(ea*ea - ma*ma);
 alpha=m2*m2-m1*m1-ei*ei+pa*pa;
```

```
 h=alpha*alpha-4.*ei*ei*m1*m1;
 more_angles:
 std::cout<< "enter theta1 in degrees"<<'\n';
 std::cin>> th1;
 th1r=th1/rad;
 csth1=cos(th1r);
 f=2.*(ei*ei-pa*pa*csth1*csth1);
 g=alpha*pa*csth1/f;
 p1 = -g + sqrt(h/2./f + g*g);
 e1 = sqrt(m1*m1+p1*p1);
 k1 = e1 - m1;
std::cout <<"p1 = "<<p1<<" th1 = "<<th1<<" k1 = "<<k1<<'\n';
// now find momentum of particle 2

 e2 = ei-e1;             // conservation of energy
 k2 = e2 - m2;
 p2 = sqrt(e2*e2-m2*m2);
 csth2=(pa-p1*csth1)/p2; // uses conservation of momentum
 th2r = acos(csth2);
 th2 = rad*th2r;
std::cout<<"p2 = "<<p2<<" th2 = "<<th2<<" k2 = "<<k2<<'\n';
std::cout <<" do another angle? y/n  ";
std::cin>> angle;
 if(angle == 'y') goto more_angles;

 return 0;
}
```

Appendix B

Example of a Monte Carlo program and the use of a random number generator

This Monte Carlo program simply finds the sum of nmax random numbers r, $0 < r < 1$. The average value $r_{avg} = sum/nmax$ should be 0.50. The program uses the c++ math library random number generator rand(). This function returns a random integer $0 < rand() < 2^{bits}$ and assumes that this is a 32 bit machine, so the largest integer possible is 2^{31}. The function rand() is called in a user defined function randf() which returns a double floating point value between 0 and 1.

In the first entry to randf() the function evaluates the normalization factor $y = 2^{31}$ and seeds the random number generator srand(time(0)). The value time(0) is the current clock time on the machine. This ensures that subsequent runs of rand() are truly random. If you want to use your own random number seed replace time(0) by an integer of your choosing.

```
// random_sum.cpp -> add together nmax random numbers between 0 and 1.
#include<iostream>
#include<cstdlib>
#include<cmath>
double randf();
using namespace std;
int main()
{
  int nmax;
  double sum;
```

```
  cout <<" Enter nmax, maximum number of random numbers ";
  cin  >> nmax;
  sum = 0.; //initialize sum to zero
  for (int i=0;i<nmax;i++)
    {
      sum=sum+randf();
    }
  sum=sum/nmax;
  cout<<" average = "<<sum<<endl;
  return 0;
}

// Introduce a user derived function randf() which calls the library
// random number generator rand(). The parameter y in randf() serves
// to bring the random number within the range 0<z<1.
double randf()
{
#include <cstdlib>
#include <ctime>
  double x,z;
  int j;
  static double y; // keep the value of y fixed from call to call of ranf()
  static int ifirst(0); // ifirst = 0 on first entry but then is reset to 1
  if(ifirst == 0) // only enter this calculation on first call to ranf()
    {
      y=pow(2.,31.); // this assumes the maximum integer for a 32 bit machine
      ifirst = 1; // we only want to calculate y on the first call
      srand(time(0)); // seed random number generator with current clock time
    }
  x=rand(); // the function rand() is part of the standard c++ math library
  z=x/y;
  return z;
}
```

Bibliography

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[4] Rexon Corp. *Internet site:http://www.rexon.com/RP408.htm.*