

California State University – Los Angeles
Department of Mathematics
Master’s Degree Comprehensive Examination
Real and Functional Analysis Syllabus
Proposed revision Spring 2002

Revision is proposed to coordinate with changes in the course requirements for the degree. The advanced calculus section and the measure and integration section remain unchanged, but the latter is now connected by an “or” to a section of topics in functional analysis

The student should be responsible for topics in part 1 and either part 2a or part 2b.

1. Advanced Calculus

Sequences and series of real numbers and functions, limit superior and limit inferior, topology of the line, continuous functions, elementary properties of the derivative.

2a. Measure and Integration

Sets and functions, countable sets, the Borel sets, sigma algebras, countably additive measures, Lebesgue measure on the line, simple functions, measurable and Borel functions, the Riemann integral, the Lebesgue integral, integral convergence theorems, convergence in measure.

2b. Functional analysis

Norms and inner products, Banach spaces and Hilbert spaces, continuous linear operators, dual spaces, weak and weak* topologies, reflexivity. The Hahn-Banach, closed graph, open mapping, and uniform boundedness principle (Banach-Steinhaus theorem). Sequence spaces, spaces of continuous functions. The Banach fixed point theorem (contraction mapping principle).

References:

- [1] H. L. Royden, *Real Analysis* 2nd ed., Macmillan Publishing Co., New York.
(Ch. 1-4)
- [2] Jerrold E. Marsden and Michael J. Hoffman, *Elementary Classical Analysis* 2nd ed., W. H. Freeman and Co., New York.
(Ch. 1-4, 5.1 - 5.2, 8.1 - 8.7)
- [3] Protter and Morrey, *A First Course in Real Analysis* 2nd ed.,
(Ch. 1-4, 5.1-5.2, 9.1-9.3)
- [4] Erwin Kreyszig, *Introductory Functional Analysis with Applications*
(Ch 1-5. (maybe part of 7 ???))

Others:

- [4] Goldberg, *Methods of Real Analysis* 2nd ed.,
(Ch. 1, 2, 3.1-3.6, 4, 5.1-5.5, 6, 7, 9.1-9.5, 11))
- [5] Cohn, *Measure Theory*
(Ch. 1-3)
- [6] Sterling K. Berberian, *Measure and Integration* , Chelsea Publishing Co.
(Ch. 1-5)