Scattered Data Approximation on the Sphere and Application to Texture Analysis

Frank Filbir

Institute of Biomathematics and Biometry
Helmholtz Center Munich

The talk deals with the approximation and optimal interpolation of functions defined on the bi-sphere $\mathbb{S}^2 \times \mathbb{S}^2$ from scattered data. We start by describing a problem which arises in crystallography and which serves as a motivation for our mathematical considerations. The problem of the determination of the so-called *orientation density function* (ODF) of a polycrystalline material is of great importance in material science. The ODF is experimentally not accessible. The measurements provide only Radon transformed and scattered data. The mathematical problem consists in the reconstruction of the ODF from these scattered data. We demonstrate how the weighted least square approximation to the function can be computed in a stable and efficient manner. The analysis of this problem is based on Marcinkiewicz-Zygmund inequalities for scattered data which we present here for the bi-sphere. The complementary problem of optimal interpolation is also solved by using good localized kernels for our setting.