

Do you Sudoku?

6		9		5	
7	4	5	6	9	
		3	6	2	
	6			5	7
9	8	2	3	5	4
	3	6		1	
		2	4	6	
6	1	7	3	8	
5		1		6	

Silvia Heubach

California State University Los Angeles
May 4, 2006

Overview

- What is Sudoku?
- Brief History
- How to play Sudoku (Strategies)
- Math Questions
- Sudoku Solvers
- Sudoku World Championship
- Sudoku Variations
- References

What is Sudoku?

- Standard Sudoku is a 9x9 grid with certain values filled in ("givens"; 24, 32 or 48)
- **Goal:** Fill remaining cells such that each row, column and 3x3 region contains the numbers 1, 2, ..., 9 exactly once
- Levels: mild, moderate, medium, devious, nasty

History

- First appeared in 1979 in New York Puzzle magazine under the name *Number Place*. Designed anonymously by [Howard Garns](#), a 74-year-old retired architect and freelance puzzle constructor
- Puzzle company Nikoli introduced it in 1984 under the name *Suuji wa dokushin ni kagiru* (= single number) in Japan

History

- In 1997, [Wayne Gould](#), a retired High Court Judge living in Hong Kong, discovered Sudoku and developed computer programs to produce Sudoku puzzles of varying difficulty
- In 2004, Gould sent puzzle to *The Times* in London, which set off a Sudoku craze
- In 2005, Sudoku spread around the world

Strategies

- Scanning (horizontal or vertical)

	3		9	4		6	
4							
			7	3		5	8
9		1		5		3	
			4		9		
		5		8		7	4
3	6		9	4			
							2
	9		1	2			7

	3		9	4		6	
4							
			7	3		5	8
9		1		5		3	
			4		9		
		5		8		7	4
3	6		9	4			
							2
	9		1	2			7

	3		9	4	x	6	x
4					x	x	x
			7	3		5	8
9		1		5		3	
			4		9		
		5		8		7	4
3	6		9	4			
							2
	9		1	2			7

	3		9	4	x	6	x
4					x	x	x
			7	3	4	5	8
9		1		5		3	
			4		9		
		5		8		7	4
3	6		9	4			
							2
	9		1	2			7

	3		9	4		6	
4							
			7	3	4	5	8
9		1		5		3	
			4		9		
		5		8		7	4
3	6		9	4			
							2
	9		1	2			7

	3		9	4		6		
4								
			7	3	4	5	8	
9	4	1		5		3		
		3	4		9			
		5		8		7	9	4
3	6	2	9	4				
							2	
	9		1	2			7	

Strategies

- Scanning (horizontal or vertical)
- Scanning combined

	3			9	4		6	
4								
				7	3	4	5	8
9	4	1	x	5	x	3		
		3	4	x	9			
		5		8		7	9	4
3	6	2	9	4				
								2
	9		1	2			7	

	3			9	4		6	
4								
				7	3	4	5	8
9	4	1	x	5	x	3		
		3	4	x	9			
		5		8	x	7	9	4
3	6	2	9	4				
								2
	9		1	2			7	

	3			9	4		6	
4								
				7	3	4	5	8
9	4	1	x	5	x	3		
		3	4	x	9			
		5	3	8	x	7	9	4
3	6	2	9	4				
								2
	9		1	2			7	

	3			9	4		6	
4								
				7	3	4	5	8
9	4	1		5		3		
		3	4		9			
		5	3	8		7	9	4
3	6	2	9	4				
								2
	9		1	2			7	

	3			9	4		6	
4								
				7	3	4	5	8
9	4	1		5		3		
		3	4		9			
		5	3	8		7	9	4
3	6	2	9	4	x			
			x		x			2
	9		1	2	x		7	

	3			9	4		6	
4								
				7	3	4	5	8
9	4	1		5		3		
		3	4		9			
		5	3	8		7	9	4
3	6	2	9	4	x			
			x	3	x			2
9		1	2	x		7		

	3			9	4		6	
4								
				7	3	4	5	8
9	4	1		5		3		
		3	4		9			
		5	3	8		7	9	4
3	6	2	9	4				
				3				2
9		1	2			7		

	3			9	4		6	7
4							3	9
		9		7	3	4	5	8
9	4	1		5		3		
		3	4		9			
		5	3	8		7	9	4
3	6	2	9	4				
				3		9	4	2
9	4	1	2			7	3	

Strategies

- Scanning (horizontal or vertical)
- Scanning combined
- Small numbers

	3			9	4	^{1 2}	6	7
4				^{1 6}		^{1 2}	3	9
		9		7	3	4	5	8
9	4	1		5		3		
		3	4	^{1 6}	9			
		5	3	8		7	9	4
3	6	2	9	4				
				3		9	4	2
9	4	1	2			7	3	

Strategies

- Scanning (horizontal or vertical)
- Scanning combined
- Small numbers
- Pairs of numbers in two cells

	3			9	4	^{1 2}	6	7
4				^{1 6}		^{1 2}	3	9
		9		7	3	4	5	8
9	4	1		5		3		
		3	4	^{1 6}	9			
		5	3	8		7	9	4
3	6	2	9	4				
				3		9	4	2
	9	4	1	2			7	3

Strategies

- Scanning (horizontal or vertical)
- Scanning combined
- Small numbers
- Pairs of numbers in two cells
- Missing values in rows, columns and region
- Using region information to exclude values in rows and columns

	3			9	4	^{1 2}	6	7
4				^{1 6}		^{1 2}	3	9
		9		7	3	4	5	8
9	4	1		5		3		
		3	4	^{1 6}	9			
		5	3	8		7	9	4
3	6	2	9	4				
				3		9	4	2
	9	4	1	2			7	3

	3	x		9	4	^{1 2}	6	7
4				^{1 6}		^{1 2}	3	9
		9		7	3	4	5	8
9	4	1		5		3		
		3	4	^{1 6}	9			
		5	3	8		7	9	4
3	6	2	9	4				
		x		3		9	4	2
	9	4	1	2			7	3

	3	x		9	4	^{1 2}	6	7
4		6		^{1 6}		^{1 2}	3	9
		9		7	3	4	5	8
9	4	1		5		3		
		3	4	^{1 6}	9			
		5	3	8		7	9	4
3	6	2	9	4				
		x		3		9	4	2
	9	4	1	2			7	3

Strategies

- Scanning (horizontal or vertical)
- Scanning combined
- Small numbers
- Pairs of numbers in two cells
- Missing values in rows, columns, region
- Using region information to exclude values in rows and columns
- "What if"

5	3	8	2	9	4	1	6	7
4	7	6	5	1	8	2	3	9
2	1	9	6	7	3	4	5	8
9	4	1	7	5	2	3	8	6
7	8	3	4	6	9	5	2	1
6	2	5	3	8	1	7	9	4
3	6	2	9	4	7	8	1	5
1	5	7	8	3	6	9	4	2
8	9	4	1	2	5	6	7	3

Math Questions

- How many 9x9 Sudoku grids are there?
- How many givens are needed to complete puzzle in a unique way?
- How many givens can there be without a unique solution?
- How does one solve a Sudoku puzzle automatically?
- What makes a Sudoku puzzle hard?

How many 9x9 Sudoku grids?

- Bertram Felgenhauer (2005) (logic and brute force computations)

$$6,670,903,752,021,072,936,960 = 9! \times 72^2 \times 27 \times 27,704,267,971$$

- Derivation simplified by Frazer Jarvis
- Independently confirmed by Ed Russel

How many 9x9 Sudoku grids?

- Felgenhauer & Jarvis (2006) (using group theory)
 - Relabeling of entries
 - Reflection; rotation
 - Permutation of blocks of columns 1-3, 4-6, 7-9
 - Permutation of blocks of rows 1-3, 4-6, 7-9
 - Permutation of columns 1-3 or 4-6 or 7-9
 - Permutation of rows 1-3 or 4-6 or 7-9
- Essentially different grids: **5,472,730,538**

Minimum number of givens

- For 9x9 Sudoku grid, lowest known is 17 givens (if no symmetry restrictions) and 18 givens if rotational symmetry is enforced
- For some of the other variants, examples with low numbers of givens have been constructed, but not for all cases.
- Only in the case of cubic Sudoku is the minimal number of givens known (=8) and examples have been constructed

Maximum number of givens

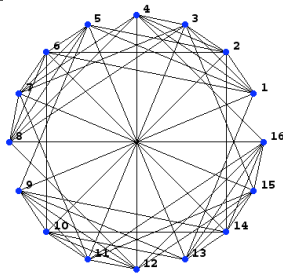
- What is the maximal number of givens for which the puzzle cannot be completed uniquely?
- Answer: **77**

	3	4	5	7		6	8	2
7	2	8	4	6	3	5	1	9
	5	6	8	2		7	3	4
8	7	9	1	5	4	2	6	3
6	4	5	2	3	8	9	7	1
2	1	3	6	9	7	4	5	8
5	8	2	3	4	6	1	9	7
4	9	1	7	8	5	3	2	6
3	6	7	9	1	2	8	4	5

Solving Sudoku Automatically

- Can express the problem as a **graph coloring problem**, where adjacent vertices have to be colored in a different color
- Each Sudoku cell becomes a vertex
- Vertices are connected if cells are in the same row, column, or region
- Givens represent pre-colored vertices
- NP-complete problem

Graph for 4x4 grid



Solving Sudoku Automatically

Efficient programs use some of the strategies we have discussed, but in a slightly different order

- Step 1:** Assign each cell all the values it can have
- Step 2:** Check if any cell has just one value; if so, assign the value and eliminate it from the cells in the same row, column and region

Solving Sudoku Automatically

- Step 3:** Check whether a particular value occurs in only one cell of a row, column or region; if so, assign this value to its cell and eliminate it from the cells in the same row, column and region. Repeat Step 2
- Step 4:** If a value occurs in only two cells in a block, and the cells are in either a row or a column, then delete the value from all other cells in that row or column. Repeat Steps 2 & 3.

Example for Step 4

(3, 9)	()	(3, 9)	()	(2, 7)	()	(3, 7)	()	()
(1, 4, 9)	(1, 4, 7, 9)	()	()	(2, 7)	()	()	(2, 9)	(2, 4)
()	(4, 5, 7)	(4, 5)	()	(7, 8)	()	(4, 7)	()	()
()	(1, 2, 5, 9)	(1, 2, 5, 9)	()	()	()	(1, 2, 5)	(1, 2, 9)	()
(1, 4, 9)	(1, 4, 5, 9)	()	()	(1, 5, 9)	()	()	(1, 9)	()
()	(2, 3, 5)	(2, 3, 5)	()	()	()	(2, 5, 7)	(2, 7)	()
()	(1, 4, 8, 9)	(1, 4, 9)	()	(1, 4, 9)	()	(1, 4, 8)	()	()
(1, 3, 4)	(1, 3, 4, 8)	()	()	(1, 4)	()	()	(1, 4, 8)	()
()	(1, 2, 4, 9)	(1, 2, 4, 9)	()	(1, 4, 9)	()	(1, 4)	()	()

Solving Sudoku Automatically

- **Step 5:** If the logic rules have been applied and no further reduction occurs, select a cell for "what if".
- **Step 6:** Try possible values until solution is found, or contradiction → *back-track*

Solving Sudoku Automatically

- **Humans:** Use logic rules first, since backtracking is hard
- **Computer:** Use only a few logic rules, since backtracking is not so hard. Important: which cell is selected for going into backtracking mode

Sudoku Competition

- First World Sudoku Championship, Lucca (Italy), March 10 - 11, 2006
- 1st place: Jana Tylova, Czech Republic
- 2nd & 3rd place: USA
- Had to solve 45 puzzles, classic + 18 variations
- 85 puzzle solvers from 22 countries
- Participants ranged from 15 to 61 years
- 1/3 were women

Sudoku Variations

- Irregular Sudoku
- Diagonal Sudoku; Extra Region Sudoku
- Cubic Sudoku
- Toroidal Sudoku
- Multiple Region Sudoku
-

References

- Robin Wilson, The Sudoku Epidemic, *Focus*, Jan 2006
- Fred Simons, Solving a Sudoku puzzle with Mathematica, *Mathematica in Education and Research*, 10:1, 2005
- http://en.wikipedia.org/wiki/Sudoku#Mathematics_of_Sudoku
- http://www.maa.org/editorial/mathgames/mathgames_09_05_05.html
- <http://www.dailysudoku.co.uk/sudoku/index.shtml>
- <http://www.afjarvis.staff.shef.ac.uk/sudoku/index.html>
- <http://www.sudoku.com/>
- <http://www.wsc2006.com/eng/championships.php>