

**California State University – Los Angeles**  
**Department of Mathematics**  
**Major Assessment Examination**  
**MATH 490    Fall 2003    Compact form**  
**Brookfield, Hoffman, Raychaudhuri**

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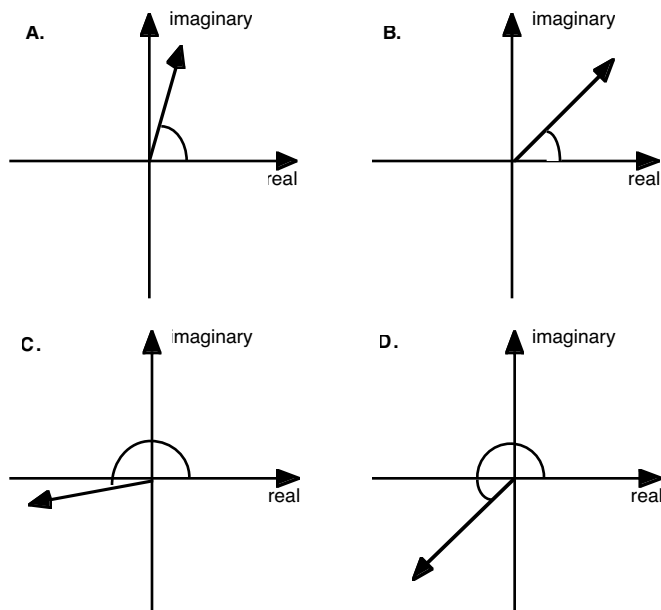
Do the following 25 problems. Problems count equally.

Problems 1-22 are multiple choice questions. Circle the letter(s) of the correct answers. (On some problems there may be more than one correct answer listed. Circle the letters of all correct answers for each problem).

A small amount of space is provided for each of these where you may provide a brief explanation of your reasons. This may help get some partial credit for an incorrect response.

Problems 23-25 are “free response” questions. For these questions you are to do what is requested – sketch a graph, give an explanation, work out a problem, etc.

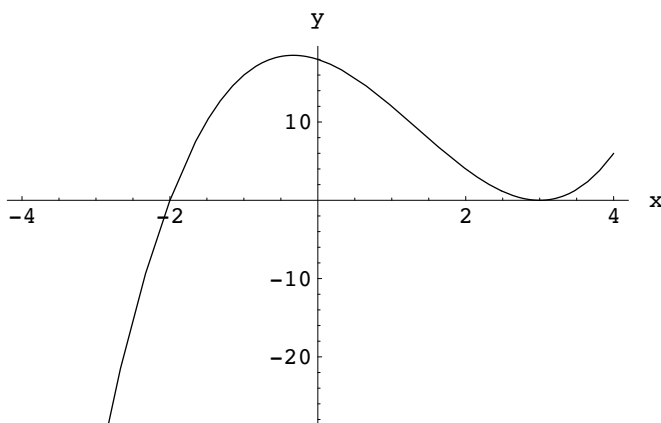
**# 1.** Given the complex number  $z = (\cos 75^\circ) + i(\sin 75^\circ)$ , which of the following best represents  $z^3$  as a vector?



# 2. If  $f(x) = \frac{e^{5x} + 6}{2}$  and  $g(f(x)) = x$  for all  $x$  in  $\mathbb{R}$ , which of the following is a formula for  $g(y)$ ?

- A.  $\frac{\ln(2y - 6)}{5}$     B.  $\frac{2y - 6}{5}$     C.  $\frac{\ln(2y - 6)}{e^5}$     D.  $\ln\left(\frac{2y - 6}{5}\right)$

# 3. Here is the graph  $y = p(x)$  of a cubic polynomial function  $p$



Which of the following statements about  $p(x)$  must be true?

- A.  $p(x)$  has at least one nonreal complex root  
 B.  $p(x)$  has a factor of  $(x - 2)$   
 C.  $p(x)$  is an odd function  
 D.  $p(x)$  has a factor of  $x^2 - 6x + 9$

# 4. An economist needs to approximate the function  $f(x) = 1/x$  by a line tangent to the graph of  $f$  at  $x = -1$ . Which of the following lines should be used?

- A.  $y = -x - 1$     B.  $y = -x - 2$     C.  $y = -2x - 3$     D.  $y = -3x - 4$

# 5. Which of the following represents the area under the curve of the function  $h(x) = \frac{4}{3x + 2}$  over the interval  $[0, 2]$ ?

- A.  $\frac{5}{16}$     B.  $\frac{15}{16}$     C.  $4 \ln 4$     D.  $\frac{4}{3} \ln 4$     E. None of these

# 6. For what values of  $x$  does the infinite series  $1 - (x - 2) + (x - 2)^2 - (x - 2)^3 + \dots$  converge?

- A.  $x > 1$     B.  $x < 3$     C.  $1 < x < 3$     D.  $x < 1$  or  $x > 3$

# 7. A function  $f(x)$  has the Taylor series  $2 + 3x + 4x^2 + 5x^3 + 6x^4 + 7x^5 + \dots$ . What is the value of  $f'''(0)$ ?

- A. 5      B.  $5/6$       C. 30      D.  $6/5$       E. None of these

# 8. Suppose  $f$  is a differentiable function whose graph passes through the points  $(1, 3)$  and  $(3, 11)$ . The mean value theorem says that there must be a point where the slope of the tangent line to the graph of  $f$  is a certain number. What is that number?

- A. 3      B. 4      C. 8      D.  $11/3$       E. None of these

# 9. Let  $f(x) = 1 - x - \sin(\pi x/2)$ . In which of the following intervals does the intermediate value theorem guarantee a solution to the equation  $f(x) = 0$ .

- A.  $[0, 1]$       B.  $[1, 2]$       C.  $[2, 3]$       D.  $[3, 4]$       E. None of these

# 10. Suppose  $f(x)$  is a differentiable function with

$$f'(x) = (x + 1)(x - 1)^2(x - 2).$$

Which of the following are true?

(Circle the numbers of all which are correct. There might be more than one.)

- A. The graph of  $f$  has horizontal tangents at  $x = -1, 1$ , and  $2$ .  
 B.  $f$  has local extremes at  $x = -1, 1$ , and  $2$ .  
 C.  $f$  has a local maximum at  $x = -1$ .  
 D.  $f$  has a local maximum at  $x = 1$ .  
 E.  $f$  has a local maximum at  $x = 2$ .

# 11. The area of the bounded region between the parabola  $y = \sqrt{x}$  and the line  $x = 2y$  is given by which of the following?

- A.  $\int_0^2 (2y - y^2) dy$       B.  $\int_0^4 (2y - y^2) dy$   
 C.  $\int_0^4 ((x/2) - \sqrt{x}) dx$       D.  $\int_0^2 (\sqrt{x} - (x/2)) dx$       E. None of these.

# 12. Each of the following equations describes a plane.

Which of them are perpendicular to the line through  $(0, 0, 0)$  and  $(1, 2, 3)$ ?

- A.  $x + 2y + 3z = 10$       B.  $x - 2y + z = 10$   
 C.  $z = x - 2y$       D.  $x + y + z = 0$       E. None of these.

# 13. How many 3 element subsets does the set  $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$  have?

- A.  $\binom{10}{3}$       B.  $3^{10}$       C.  $2^{10}$       D.  $10^3$       E. None of these

# 14 The coefficient of the term  $x^8y^3$  in the expansion of  $(2x - y)^{11}$  is

- A.  $2^8$     B.  $-2^{11}$     C.  $\binom{11}{2}2^8$     D.  $\binom{11}{8}2^8$     E.  $-\binom{11}{3}2^8$

# 15. What is the dimension of the subspace  $W$  of  $\mathbb{R}^4$  described by

$$W = \{(x, y, z, t) \in \mathbb{R}^4 : x + y + z + t = 0 \text{ and } x + 2y + z + 2t = 0\}?$$

- A. 0    B. 1    C. 2    D. 3    E. 4

# 16. An equivalence relation is defined on the set  $\mathbb{Z}$  of integers by  $n \cong k$  if the difference,  $n - k$ , is divisible by 3. How many equivalence classes are there?

- A. 1    B. 2    C. 3    D. 4    E. Infinitely many

# 17. Let  $H = \langle 12 \rangle$  be the subgroup of  $(\mathbb{Z}_{30}, +)$  generated by 12. What is the order of the element  $3 + H$  in the group  $\mathbb{Z}_{30}/H$ .

- A. 2    B. 3    C. 4    D. 5    E. 6

# 18. Which of the following cycles are in the subgroup of  $S_5$  generated by the cycle  $(1, 2, 3, 4)$ ? (Circle the letters of all which are correct.)

- A.  $(2, 4, 3, 1)$     B.  $(3, 4, 1, 2)$   
 C.  $(3, 2, 1, 4)$     D.  $(2, 3, 4, 5)$     E. None of these

# 19. Consider a set  $S = \{a, b, c, d\}$  with an associative operation  $*$  given by the table

*	a	b	c	d
a	b	d	a	d
b	d	d	b	d
c	a	c	c	d
d	d	d	d	d

(Circle the letters of all of the following which are correct.)

- A.  $*$  is a commutative operation on  $S$ .  
 B.  $S$  contains an identity element (with respect to  $*$ ).  
 C. The inverse of  $a$  is  $b$ .    D.  $(S, *)$  is a group.    E. None of these

# 20. Suppose  $q_1, q_2, q_3, \dots$  is an infinite sequence of rational numbers with  $0 < q_n < 100$  for all  $n$ . Which of the following must be true? (Circle the letters of all that are true. There may be more than one.) (Cluster points are sometimes called accumulation points.)

- A. The sequence must converge.  
 B. The sequence must have a cluster point in  $\mathbb{R}$ .  
 C. The sequence must have a rational cluster point.  
 D. The sequence must have a cluster point  $p$  with  $0 < p < 100$ .  
 E. The sequence must have a Cauchy subsequence.

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# 21. For  $n = 1, 2, 3, 4, \dots$  let  $a_n = (-1)^n + \frac{1}{n}$ . Let  $S = \{a_1, a_2, a_3, a_4, \dots\}$   
What are  $\inf S$  and  $\sup S$ ?

- A.  $\inf S = -1$  and  $\sup S = 1$       B.  $\inf S = 0$  and  $\sup S = 3/2$   
 C.  $\inf S = -1$  and  $\sup S = 3/2$       D.  $\inf S = -2/3$  and  $\sup S = 3/2$   
 E.  $\inf S = 3/2$  and  $\sup S = -1$

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# 22. Which of the following are open subsets of  $\mathbb{R}$ ?

(Circle the letters of all that are open, there may be more than one.)

- A.  $\{x \in \mathbb{R} : x^2 \leq 1\}$       B.  $\{x \in \mathbb{R} : x^2 < 1\}$       C.  $\{x \in \mathbb{R} : x^2 \leq -1\}$   
 D.  $\bigcap_{n=3}^{\infty} \{x \in \mathbb{R} : -1/n < x < 1/n\}$       E.  $\bigcup_{n=3}^{\infty} \{x \in \mathbb{R} : 1/n < x < 1 - 1/n\}$

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# 23. Sketch the graph of a function with all of the following properties

- (i)  $f(x) < 0$  for  $x < 0$       (ii)  $f'(x) < 0$  for  $x < 0$   
 (iii)  $f$  has a vertical asymptote at  $x = 0$  (two-sided)  
 (iv)  $f'(x) > 0$  for  $0 < x < 2$       (v)  $f(2) = 2$   
 (vi)  $f'(x) < 0$  for  $x > 2$       (vii)  $f(x) > 0$  for  $x > 1$

( A grid was supplied for drawing the graph.)

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# 24. Let  $\vec{v}_1 = (1, 2, 1)$  and  $\vec{v}_2 = (0, 2, 1)$  be vectors in  $\mathbb{R}^3$ .  
Find a vector  $\vec{v}_3$  in  $\mathbb{R}^3$  such that  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$  is a basis for  $\mathbb{R}^3$ .  
Explain how you know that it is a basis.

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# 25. Explain what is wrong with the following “proof” by mathematical induction (other than the fact that the proposition is false)?

**Proposition.** For every positive integer  $n$ , the number  $3^n$  is even.

PROOF: Make the induction hypothesis that the assertion is true for  $n = k$ . Then  $3^k$  would be even, so there would be an integer  $j$  with  $3^k = 2j$ .

Using this induction hypothesis, we can compute that for  $n = k + 1$ ,

$$3^n = 3^{k+1} = (3)(3^k) = 3 \cdot (2j) = 2 \cdot (3j).$$

The number  $3^n$  would be a multiple of 2 and thus be even.

Truth of the proposition for  $n = k$  implies its truth for  $n = k + 1$ . Thus the proposition is true for all positive integers  $n$  by the principle of mathematical induction. ■

EXPLANATION:

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## End of Exam